Journal of Finance & Economics
Volume 2, Issue 2 (2014), 16-53
ISSN 2291-4951 E-ISSN 2291-496X
Published by Science and Education Centre of North America

Securitization under Asymmetric Information and Risk Retention Requirement

Haejun Jeon[1*] & Michi Nishihara[1]

[1] Graduate School of Economics, Osaka University, Osaka, Japan

*Correspondence: Haejun Jeon, Graduate School of Economics, Osaka University, 1-7 Machikaneyama-cho, Toyonaka, Osaka 560-0043, Japan. Tel: 81-6-6850-5277. E-mail: haejun.jeon@gmail.com

DOI: 10.12735/jfe.v2i2p16

Abstract

We address a three-period model of financial intermediaries that involves securitization of risky loan assets and asymmetric information. We show that the risk retention requirement with a fixed ratio, stipulated by the Dodd-Frank Act, might induce losses of social welfare in the sense that a bank might not utilize profitable investment opportunities due to the regulation, which leads to a downward jump in social welfare. We present various structures of social welfare with respect to the level of 'skin in the game', and clarify the necessity of countercyclical regulation by verifying that the social losses caused by current regulation become more severe during a recession. Furthermore, we verify how the financial market becomes volatile through securitization and leverage.

JEL Classifications: G21, G24, G28

Keywords: securitization, risk retention requirement, skin in the game, asymmetric information, countercyclical regulation

1. Introduction

The Securitization, a process to convert illiquid loans into liquid securities, has been playing a significant role in financial markets since it emerges in the 1970's. It has lubricated the markets and increased optimal risk sharing among economic subjects in the sense that firms have been able to reduce their funding costs and financial intermediaries have been able to lower capital requirements by virtue of securitization, and for these reasons, the market of securitization has grown explosively for decades. However, the financial crisis hit in 2007, and the issuance of securitized assets plummeted after the bubble burst,[1] and furthermore, it has been criticized and condemned as the main culprit of the financial crisis in 2007. After thorough inspection regarding the cause of the financial crisis, the authorities in the United States and Europe proposed financial regulation that requires financial intermediaries to disclose more information, limits proprietary trading and investment of commercial banks ('Volcker rule'), and forces financial institutions to retain a certain amount of credit risks ('skin in the game').[2]

---

[1] In the United State, for instance, it dropped from $2.147 trillion in 2007 to $933 billion in 2008. Refer to AFME (2011) for specific data.

[2] Refer to Subtitle D of Title IX of the Dodd-Frank Wall Street Reform and Consumer Protection Act for specific information about regulations on asset-backed securitization. The Dodd–Frank Wall Street Reform and Consumer Protection Act were enacted on July 21, 2010 and is expected to affect almost every financial institutes in the United States.

Especially, the European Union (EU) Parliament and the government of the United States stipulated 5% of uniform mandatory risk retention.[3] The regulation, however, takes the form of a 'vertical' slice with a 'fixed ratio', and has thus been criticized in recent papers. Mostly, the criticism focuses on the retention of a 'vertical slice' which might not be optimal to incentivize financial intermediaries to monitor borrowers (e.g. Fender & Mitchell, 2009; Kiff & Kisser, 2010). Our research also points out the flaw of compulsory risk retention, but we pay attention to other aspects of the current regulation. We raise a question regarding the efficiency of risk retention with a 'fixed ratio' applied to every financial institution uniformly, without considering features of individual intermediary or business cycle. This doubt is in line with recent literature (e.g. Dugan, 2010; Wu & Guo, 2010; Batty, 2011; Levitin, 2011).

We address a model of financial intermediaries with securitization based on Shleifer and Vishny (2010). We improve their model in various ways, not losing any significant features of the original model. First of all, we introduce risky real investment projects. In Shleifer and Vishny (2010), there is no fundamental risk in real investment projects, and volatility of the financial market stems from investors' sentiment. Our model does not involve any kind of irrationality and can still derive novel results. This is made possible by introducing asymmetric information between the bank and investors and other real investment projects stochastically available at the interim periods. These presumptions are plausible considering that issuers of securitized assets actually have private information, and investment projects might be available in the future but we cannot say for sure as of now. If there asymmetric information exists between the issuer and investors about return of the initially issued assets and availability of the new projects in the interim period, adverse selection problem occurs regarding the securitized assets.

In our model, social welfare is evaluated as the sum of expected return of the bank and investors, and is thus affected by the level of skin in the game. A jump, however, might occur in social welfare due to information asymmetry and the bank's profit maximization. A downward jump implies that the bank does not utilize profitable projects fully, which is a severe loss of social welfare. Not only might the structure with one jump, either downward or upward, but also with both jumps occur. We categorize the structure of social welfare and present the conditions for each type to hold and numerical examples.

Based on these arguments, we clarify that the regulation of risk retention could aggravate the securitization market.[4] The side effects of the regulation could occur when the fixed ratio of risk retention stipulated by the government is higher than the threshold of the downward jump in social welfare. On the other hand, if the regulated level of skin in the game is slightly lower than the threshold of an upward jump, tightening the regulation a little more would improve social welfare remarkably. Furthermore, we verify that the possible loss of social welfare gets more severe during recession. That is, the bank is less likely to utilize the new projects when expected return and availability of the new projects are low. In economic downturn, a threshold of skin in the game regarding the downward jump in social welfare gets lower, and the depth of the downward jump gets larger while that of the upward jump gets smaller. These results imply that a tight regulation regarding risk retention could induce more severe loss of social welfare in recession, which shed light on the necessity of the countercyclical regulation regarding risk retention.

If the bank is levered and the lemon problem occurs, it might have to liquidate a fraction of its assets on balance sheet to keep a haircut level, even though the actual quality of assets is high; this leads to a volatile financial market, which is explained depending only on investors' sentiment in Shleifer and Vishny (2010).

---

[3] Refer to IMF (2009).

[4] Dugan (2010) stated that "a requirement intended to improve the securitization market by improving the quality and trustworthiness of underwriting could significantly curtail the number of securitizations."

Literature related to our research is presented in Section 2. In Section 3, we illustrate the basic framework of our model, such as investment projects and information structure. We provide an equilibrium analysis for when the information structure is given exogenously in Section 4.1, and for when it is determined endogenously in Section 4.2. Analysis regarding social welfare and the level of risk retention is presented in Section 5; various types of social welfare structures, comparative statics, and implications regarding the risk retention requirement are presented. Section 6 extends the basic model to the levered bank and shows that we can still verify how the financial market becomes volatile. Section 7 summarizes the arguments of this paper.

2. Related Literature

After the official announcement of financial regulation intended to stabilize the financial system and protect consumers from abusive financial services, many researchers have investigated adequacy and efficiency of the current regulation. Fender and Mitchell (2009) assert that the retention of the vertical slice might be suboptimal in terms of incentives to monitor borrowers and that intermediaries' incentives to monitor borrowers depend not only on the return of assets but also on the 'thickness' of tranches. Kiff and Kisser (2010) extend Fender and Mitchell (2009) to attain an optimal retention size explicitly. Wu and Guo (2010) show that a flat ratio of risk retention yields worse results and claim that the requirement of information disclosure, rather than that of risk retention, is more efficient. Dugan (2010) alleges that if off-balance sheet treatment is crucial for revitalizing the securitization market, the risk retention requirement will worsen the situation, and recommends to establish a minimum underwriting standard rather than to compel banks to retain fixed risks. Kiff and Kisser (2011) provide abundant numerical analysis, implications, and extensions based on their earlier work (2010). Batty (2011) insists that the new issuance of CLOs, different from CDOs in the sense that most of them are actively managed by third party managers, might shrink due to the current regulation. Levitin (2011) addresses that the moral hazard problem of credit card securitization can be resolved by implicit recourse, not by risk retention.

In this paper, we limit ourselves to verifying the possible side effects of the risk retention with fixed ratio from various angles, but numerous studies have attempted to explore the optimal amount of risk retention and the optimal structure of subordination under asymmetric information. Riddiough (1997) analyzes the design and governance of risky asset-backed securities with asymmetric information and liquidation motives. DeMarzo and Duffie (1999) assert that downward-sloping demand curve might exist due to the issuer's private information, and investigated optimal security design based on a tradeoff between retention costs and liquidity costs. DeMarzo (2005) verifies why pooling and tranching of securities is beneficial for issuers if there asymmetric information exists. Gorton and Souleles (2007) address the benefit of special purpose vehicles in the process of securitization. Hartman-Glaser, Piskorski and Tchistyi (2012) develop an optimal design of mortgage-backed securities in a continuous model. Malekan and Dionne (2012) illustrate a model that endogenously specifies the exact form and amount of optimal retention.

There exist papers that examine why modern banking is unstable and vulnerable to shocks. Shleifer and Vishny (2010) provide a model that involves securitization and leverage and show that a levered bank is inherently volatile. Gennaioli, Shleifer, and Vishny (2010) introduce financial innovation and show that financial intermediaries can be volatile and fragile, even without leverage, by assuming that investors neglect certain unlikely risks and demand securities with safe cash flows.

Some articles discuss intervention of the authorities for financial stability. Diamond and Rajan (2011) doubt whether the authorities have to clean up a banking system by closing some banks and forcing others to liquidate assets if the crisis seems to occur. Stein (2012) explains why unregulated private money creation leads to unstable market, why supplementary policy other than open-market operations is necessary in more advanced economies, and how monetary policy affects bank lending and real activity.

3. Model Setup

Our model looks similar to that of Shleifer and Vishny (2010), but the focus is quite different. The central aim of this paper is to show the risk retention requirement might yield severe side effects in terms of social welfare.[5]

3.1 Bank, Firms, and Investors

There exist firms that have real investment projects with i.i.d. stochastic return and do not have their own capital. The only source of financing is borrowing from banks.[6] Banks do not differ from each other in terms of costs of capital and private information, and thus, it is possible to consider a representative bank.

Outside investors are key players in the securitization market for several reasons. First, they participate in market making by purchasing securitized assets with perfect inelasticity. Second, they function as liquidity providers, i.e., lenders to the bank, as we introduce leverage in Section 6. We assume that investors can participate in financing the projects only through the bank's borrowing. In other words, investors cannot lend to the firms directly due to a lack of information and high costs of monitoring. They need not be risk neutral. Surplus from investment projects is split between the bank and investors, depending on their bargaining power, but it is not precisely a 'risk premium' as it does not reflect the amount of risk in the projects.

3.2 Timeline

Our model adopts the stylized three-period model, and timeline consists of period 0, 1, and 2.

At time 0, the bank invests in the firm's projects, which are always available in the initial period, utilizing its equity capital. There is no asymmetric information at this time.

The payoff of the projects undertaken at the initial period is revealed at time 1. If there asymmetric information exists between the bank and investors, the return is only known to the bank. Furthermore, another real investment projects might exist at time 1. If the structure of information is asymmetric, the bank can only identify whether the new projects are available. At time 0, however, neither the bank nor investors can observe whether new projects are available at time 1. Thus, asymmetric information takes place at time 1, if it exists at all. If the bank borrows from outside investors in the initial period and thus holds liabilities, mandatory liquidation that makes the financial market volatile might occur at time 1, depending on the market value of securitized assets. This extension will be discussed in Section 6.

The information asymmetry is resolved at time 2. Payoff of both real investment projects, undertaken at time 0 and time 1, are realized at terminal period, and thus, profits and losses of both the bank and investors are also realized.

3.3 Investment Projects and Securitization

We denote the investment opportunity always available at the initial period I_X and its stochastic payoff X. The supply of I_X investment projects is infinite. That is, the bank could utilize these projects as much as it wants if it has enough capital to invest. Each project costs \$1 to undertake at the initial period and yields stochastic payoff X at the terminal period. It has a positive net present value ($\mathbb{E}[X] > 1$), and the payoff X follows Bernoulli distribution. High payoff and low payoff,

[5] Nonetheless, the paper illustrates what Shleifer and Vishny (2010) asserted via an even more realistic setup without the assumption of investor sentiment if leverage is included, which is shown in Section 6.

[6] An alternative explanation for this assumption is that screening or monitoring by an informed intermediary is required to distinguish the projects with a positive net present value from those with a negative one.

denoted by $X_H > 1$ and $X_L < 1$, respectively, are realized with probability of p and $1-p$, respectively. Because all projects do not have to be identical, $100 \cdot p$ % of projects are successful whereas $100(1-p)$% of them are not.

For traditional lending, a bank simply originates loan contracts; in modern banking, it securitizes loan assets and distributes them to the market. The bank keeps d, called 'skin in the game', a fraction of the projects on its balance sheet, and sells others through special purpose vehicle (SPV). The ratio is determined by a consensus of lenders and borrowers. After the financial crisis in 2007, however, the authorities recognized the problem of reckless securitization pursued by financial institutions, and stipulated a fixed ratio of risk retention in the United States and Europe. We further assume that the bank need not hold the securities for more than one period as assumed by Shleifer and Vishny (2010).[7]

Another investment opportunity, denoted by I_Y, is available at the interim period with ex ante probability q. Neither the bank nor investors can observe precisely at the initial period whether I_Y projects are available or not. The supply of I_Y projects is also assumed to be infinite, and it costs \$1 to invest with stochastic payoff Y realized at terminal period. It also has positive net present value (i.e. $\mathbb{E}[Y] > 1$), but we do not need any assumption about the distribution of Y except that Y is integrable. This additional investment project plays a crucial role in our analysis, and lets us have a number of novel results. Parlour and Plantin (2008) also adopt analogous assumption. They consider stochastic discount factor, stating that it captures the idea that the bank receives new private opportunities, but may not be able to seize them due to illiquid market or binding regulation regarding capital adequacy.

If the bank expends all of the capital to take advantage of I_X projects at the initial period, it has to sell the assets on balance sheet to utilize the new projects at the interim period as it does not have any cash assets at that time. At this point, adverse selection problem arises (Akerlof, 1970). Whether market failure occurs depends on the distribution of payoff from I_X, expected return of I_Y projects, and the ex ante probability that I_Y becomes available. This lemon problem makes it possible to induce a volatile financial market, which will be discussed in Section 6. Shleifer and Vishny (2010) addressed this development by assuming investors' sentiment.

We presume that proceeds from investment projects are distributed to banks and investors who take on the risks of the projects. In this competitive market, firms make no profit, and social welfare is the sum of profits of banks and investors.[8] Banks take $\alpha \in [0,1]$ and $\beta \in [0,1]$ portion of proceeds from I_X and I_Y, respectively, while investors take $1-\alpha$ and $1-\beta$ portions of each project, respectively. These parameters are determined by their bargaining power. Shleifer and Vishny (2010) also assume that entrepreneurs and banks split the surplus from the projects, but fundamental risks, for which investors might ask for risk premium, do not exist in their model. In contrast, there certainly exist fundamental risks in the projects in our model, and thus, it is plausible to presume that not only banks but also investors ask for proceeds from the projects. Furthermore, we suppose that the bargaining power depends on the information they have. That is, those who have private information hold a dominant position in their bargaining power. Detailed explanation will be provided in the following subsections.

3.4 Information Structure

As mentioned in the previous subsections, we consider two levels of asymmetry of information between the bank and investors. One is about the realized payoff of I_X projects and another is

[7] This could mean that the purpose of regulation regarding skin in the game is to limit the number of securitized assets distributed in the markets.

[8] This assumption does not cause any significant problem as we focus on the dynamics between the bank and investors.

about the availability of the new projects, I_Y, and both of them occur at the interim period. In other words, the bank can observe X and whether I_Y is exploitable at time 1 while outside investors cannot. It is also assumed that the bank cannot send a signal about availability of I_Y investment opportunity to outside investors.

We further assume that those who have private information get the upper hand on those who do not when they split surplus from investments. That is, $\beta_a > \beta_s$ holds where β_a and β_s denote a fraction of surplus the bank gets from I_Y projects under asymmetric and symmetric information, respectively. Yet, α is not affected by the information structure since there is no information asymmetry at the initial period at which the contract α is involved is made. Rajan (1992) adopts an analogous idea in a model where firms could finance through either informed banks or arm's length investors, and informed banks have more bargaining power and require more surplus than arm's length investors. Wu and Guo (2010) also argue that securitization under asymmetric information between banks and investors leads to structural distortion of social welfare, which implies that expected utility is transferred from investors to the bank, which has private information.

Shleifer and Vishny (2010) assume that there is no conflict of interest between bank shareholders and creditors, so the alignment of the bank's profitability and social efficiency holds unless there exist bubbles at the initial period. In our model, this is not the case if β_s and β_a differ from each other. The following section will show this with straightforward calculations and figures. The discordance of maximization of profits for those who have private information and social welfare is a universal phenomenon.

We analyze equilibrium through an exogenously fixed information structure in Section 4 and through an endogenously determined information structure in Section 5. While analyzing equilibrium under endogenous information structure, we postulate that the bank can choose to disclose its private information, having considered the benefit and the cost of private information, and this analysis leads to novel results regarding the inefficiency of compulsory risk retention.

4. Equilibrium Analysis

4.1 Exogenously Given Information Structure

4.1.1 *Symmetric Information*

By symmetric information structure, we refer to the situation in which not only the bank but also the investors can observe the realization of X precisely and know whether I_Y projects are available at time 1. In this case, the lemon problem regarding sales of assets, of which payoff is X, does not take place, and the bargaining power of the bank with respect to splitting the surplus from the new projects is low.

The bank has equity capital of E_0 at time 0. It can use the capital to lend to I_X projects, or hold it as cash, denoted by C. The amount of cash hoarding is either $C = 0$ or $C = E_0$ because of the linearity of the expected profit function. The condition for the bank not to hoard cash is provided in Appendix A, and it always holds when information is symmetric.

In traditional lending, the bank simply invests E_0 in the projects. In modern banking, however, the bank not only originates loans but also distributes them. That is, it sells them through a special purpose vehicle (SPV). Let P_t denote the price of securitized assets regarding I_X projects at time t. Because investors take $(1-\alpha)$ fraction of surplus from I_X projects, the initial price P_0 is determined as follows:

$$P_0 = \mathbb{E}(X) - (1-\alpha)[\mathbb{E}(X) - 1] = 1 + \alpha[\mathbb{E}(X) - 1] =: \overline{P}. \tag{1}$$

After selling the securitized assets at this price, the bank will use the capital it receives from selling the securitized assets to invest in the same project repeatedly. For tractability, we assume that the bank immediately distributes the profit from the sale, $\alpha[\mathbb{E}(X)-1]$, as dividends or compensation for employees, and utilizes only 1 to invest. This assumption can be explained by competitive pressures in the banking industry, which is also adopted by Shleifer and Vishny (2010).

The bank securitizes in this manner and keeps $d \in (0,1]$ fraction of the entire projects on its balance sheet.[9] By doing this, both the number of investment projects undertaken and the expected return from them become $1/\mathrm{d}$ times of those without securitization. In this regard, we can state that alignment of social welfare and the bank's profit holds under symmetric information, and well-driven securitization augments both social welfare and the bank's profit.

The ratio d, called 'skin in the game', is usually determined by the consensus of participants in the contract, considering the principle-agent problem. Arguments supporting this position can be found in Gorton and Pennacchi (1995), and we do not evaluate the endogenous value of d in this paper. We rather assume that there exists a lower bound of d, denoted by $\underline{d}$, due to the principal-agent problem or technical issue, and shed light on the side effect of regulation stipulated by the authorities, denoted by $\overline{d}(>\underline{d})$.

The ex ante probability that I_Y projects become available at time 1 is q. If they become available, the bank sells the assets on balance sheet at the price of the realized payoff to exploit the new investment opportunity. The bank not only originates the new loans but also distributes them in the same manner as I_X projects. With probability $1-q$, however, no other investment opportunity is available at time 1. In this case, the return from each project is $X_H-1>0$ and $X_L-1<0$ with probability p and $1-p$, respectively. This is different from Shleifer and Vishny (2010) in which the one who holds assets until the terminal period always receives a positive payoff, i.e., there is no fundamental risk in their model.[10]

As explained in the previous section, surplus from the projects is split between the bank and investors based on their bargaining power, denoted by α and β_s for I_X and I_Y projects, respectively. Because the bank holds d fraction of total projects as a skin in the game, the bank's expected return includes not only that as an issuer of securitized assets but also that as an investor. Thus, the bank's expected return at time 0 is as follows:

$$\alpha[\mathbb{E}(X)-1]\frac{E_0}{d}+(1-\alpha)[\mathbb{E}(X)-1]E_0$$

$$+q\{\beta_s[\mathbb{E}(Y)-1]\frac{\mathbb{E}(X)E_0}{d}+(1-\beta_s)[\mathbb{E}(Y)-1]\mathbb{E}(X)E_0\}. \tag{2}$$

The first two terms in (2) are related to I_X projects, of which a contract is made at the initial period. Among them, the first one is expected profit as an issuer — that is, from all securitized assets the bank distributes — and the second one is that as an investor who takes the same risks as other outside investors — that is, from assets on the balance sheet. The terms in the second row of (2) are related to I_Y projects available at time 1 with ex ante probability q. The same argument is applied to explain the composition of the terms in a brace. Available capital, however, is different from that at the initial period as the market value of assets regarding I_X projects changes depending on the state of nature at the interim period. It becomes $X_H E_0$ or $X_L E_0$ with probability p and $1-p$, respectively.

Similarly, expected profit of investors at time 0 is as follows:

[9] $d=1$ corresponds to traditional lending in which securitization is not undertaken.

[10] Nevertheless, the price of assets falls down during the interim period due to investors' sentiments.

$$(1-\alpha)[\mathbb{E}(X)-1](\frac{1}{d}-1)E_0+q\{(1-\beta_s)[\mathbb{E}(Y)-1](\frac{1}{d}-1)\mathbb{E}(X)E_0\}. \tag{3}$$

The amount of capital affected by the return of assets is $(1/\mathrm{d}-1)E_0$, not E_0/d, because E_0 is already reflected in (2) as the amount of assets on the bank's balance sheet.

Social welfare, the sum of (2) and (3), is as follows:

$$[\mathbb{E}(X)-1]\frac{E_0}{d}+q\{[\mathbb{E}(Y)-1]\frac{\mathbb{E}(X)E_0}{d}\}. \tag{4}$$

Social welfare, of course, does not depend on α and β_s, the bargaining power between the bank and investors. If it were not for securitization, social welfare would be d times (4), as follows:

$$[\mathbb{E}(X)-1]E_0+q\{[\mathbb{E}(Y)-1]\mathbb{E}(X)E_0\}. \tag{5}$$

4.1.2 *Asymmetric Information*

Now we illustrate how the results change under asymmetric information. When the information structure is asymmetric, only the bank knows the realization of X and availability of I_Y projects at time 1, and the bargaining power of the bank will increase from β_s to β_a because of the private information it has. In other words, the increase from β_s to β_a can be interpreted as the bank's benefiting from private information. The discordance of β_s and β_a leads to a misalignment of the maximization of the bank's profit and that of social welfare. Furthermore, adverse selection problem of sales of assets at the interim period occurs. The price of securitized assets that the bank is willing to sell depreciates, and we denote the depreciated price P^*. If the new investment opportunity is lucrative enough, the bank would sell all assets on the balance sheet to utilize I_Y projects regardless of the payoff realized from I_X projects, and this occurs with ex ante probability q. If I_Y projects are not available, the bank would sell its assets only when X_L is realized. Thus, the depreciated price is as follows:[11]

$$P^*=q\overline{P}+(1-q)X_L. \tag{6}$$

If the bank sells assets at the depreciated price P^* even though X_H is realized from I_X projects, a loss occurs from the sale. For the bank to enforce the sale in spite of the loss, expected profit from the new investment has to exceed the amount of the losses induced by the lemon problem. Otherwise, the bank will sell its assets only when X_L is realized, and this leads to market failure in which only assets with poor quality are traded.[12] This argument can be expressed as follows:

$$[X_H-P^*]E_0\le\beta_a[\mathbb{E}(Y)-1]\frac{P^*E_0}{d}+(1-\beta_a)[\mathbb{E}(Y)-1]P^*E_0. \tag{7}$$

which can be arranged with respect to d as follows:

$$d\le\frac{\beta_a[\mathbb{E}(Y)-1]P^*}{X_H-P^*-(1-\beta_a)[\mathbb{E}(Y)-1]P^*}=:d^*. \tag{8}$$

---

[11] For tractability, we suppose that investors are risk neutral about the risks of I_Y projects' availability. If this is not the case, another parameter regarding bargaining power has to be added.

[12] Analogous argument can be found in Plantin and Parlour (2008). They distinguish liquid secondary market, in which both failed and successful claims are traded, from illiquid secondary market, in which only failed claims are sold.

The condition (8) is more likely to hold as $\mathbb{E}(Y)$, q, and β_a get higher. That is, the better the new investment opportunity is, more likely the condition is to hold. If the condition does not hold, the bank would sell only if X_L is realized, and thus, $P_1 = X_L$. The condition (8) plays a crucial role in our analysis, especially in Section 5.[13]

Meanwhile, if (8) holds and the bank sells assets at the depreciated price P^*, the expected profit from assets on the balance sheet decreases, as mentioned above. To examine this in detail, let us recall the bank's expected return from assets related to I_X projects on the balance sheet. Investors buy the securitized asset at the price $P_0 = \overline{P}$, and the payoff of the asset is X_H and X_L with probability p and $1-p$, respectively. Thus, expected return from each asset is asfollows:

$$pX_H + (1-p)X_L - P_0 = \mathbb{E}(X) - \overline{P} = (1-\alpha)[\mathbb{E}(X) - 1]. \tag{9}$$

If the bank determines to sell the assets at the depreciated price P^* whenever the new projects are available regardless of the payoff realized from I_X projects, the expected return from each asset on balance sheet is as follows:

$$P^* - P_0 = \{q\overline{P} + (1-q)X_L\} - \overline{P} = -(1-q)(\overline{P} - X_L). \tag{10}$$

This occurs when I_Y projects are available, which happens with ex ante probability q. If they are not available, the bank would sell the assets only when X_L is realized. If X_H is realized, the bank would keep them until the terminal period.[14] Hence, expected return as an investor from each I_X asset in this case is as follows:

$$p(X_H - P_0) + (1-p)(P^* - P_0) = P^* - \overline{P} + p(X_H - P^*). \tag{11}$$

Thus, the total expected return from each asset on the balance sheet, by straightforward calculation given in Appendix C, is as follows:

$$\begin{aligned} q(P^* - P_0) + (1-q)\{p(X_H - P_0) + (1-p)(P^* - P_0)\} \\ = (1-\alpha)[\mathbb{E}(X) - 1] - T_1 \end{aligned} \tag{12}$$

where

$$T_1 := q[\{1 - p(1-q)\}(1-\alpha)[\mathbb{E}(X) - 1] + p(1-q)[\mathbb{E}(X) - X_L]].$$

The difference between (9) and (12), T_1, is a loss of the bank's expected return due to information asymmetry, and it is transferred from the bank to investors, and thus, social welfare does not change. In spite of the transfer, the bank might prefer information asymmetry to symmetry because the expected return from new projects increases when the bank has private information for $\beta_a > \beta_s$.

Meanwhile, the no cash hoarding condition provided in Appendix A might not hold if information is asymmetric because of the loss from the transfer of expected return and mandatory liquidation. We only consider the situation in which no hoarding condition holds hereafter.

Suppose that (8) holds. Then the bank will sell assets on the balance sheet regardless of the realization of X whenever the new projects are available, so $P_1 = P^*$. In this case, expected profit of the bank at time 0 is as follows:

[13] The value of d^* becomes a criterion of a downward jump in social welfare, which might occur due to the adverse selection problem. A detailed explanation will be presented in Section 5.

[14] If the bank has liabilities, it might have to liquidate a fraction of them for a haircut. This will be discussed in Section 6.

$$\alpha[\mathbb{E}(X)-1]\frac{E_0}{d}+(1-\alpha)[\mathbb{E}(X)-1]E_0-T_1E_0$$
$$+q\{\beta_a[\mathbb{E}(Y)-1]\frac{P^*E_0}{d}+(1-\beta_a)[\mathbb{E}(Y)-1]P^*E_0\}. \tag{13}$$

A portion of expected profit, T_1E_0, is transferred to investors, as explained above, and the amount of capital for the new investment is decreased from $\mathbb{E}(X)E_0$ to P^*E_0. However, (13), the bank's expected profit under asymmetric information, can exceed (2), that under symmetric information as long as β_a is significantly higher than β_s. If this is the case, the bank benefits from information asymmetry.

Meanwhile, the investor's expected profit at time 0 is as follows:

$$(1-\alpha)[\mathbb{E}(X)-1](\frac{1}{d}-1)E_0+T_1E_0$$
$$+q\{(1-\beta_a)[\mathbb{E}(Y)-1](\frac{1}{d}-1)P^*E_0\}. \tag{14}$$

A fraction of expected benefit, T_1E_0, is transferred from the bank to investors, but they suffer a loss from decrease in bargaining power and the amount of capital invested in I_Y projects.

Social welfare, the sum of (13) and (14), is as follows:

$$[\mathbb{E}(X)-1]\frac{E_0}{d}+q\{[\mathbb{E}(Y)-1]\frac{P^*E_0}{d}\}. \tag{15}$$

This is strictly lower than (4), social welfare under symmetric information due to $P^*<E(X)$. In brief, social welfare decreases due to information asymmetry, while the bank benefits from it if β_a is significantly higher than β_s.

Now suppose that (8) does not hold. In this case, the bank will liquidate assets on its balance sheet only when X_L is realized, and thus, $P_1=X_L$. In other words, the bank does not utilize a lucrative investment opportunity when X_H is realized. This is a fatal loss of social welfare incurred by information asymmetry.[15]

Expected profit of the bank at time 0 is as follows:

$$\alpha[\mathbb{E}(X)-1]\frac{E_0}{d}+(1-\alpha)[\mathbb{E}(X)-1]E_0$$
$$+q(1-p)\{\beta_a[\mathbb{E}(Y)-1]\frac{X_LE_0}{d}+(1-\beta_a)[\mathbb{E}(Y)-1]X_LE_0\}. \tag{16}$$

The differences between (16), with the condition not satisfied, and (13), when the condition is satisfied, not only include the fact that P^* is substituted for X_L but also that $(1-p)$ is multiplied by the terms in a brace.[16]

Expected profit of investors also decreases if (8) does not hold as follows:

$$(1-\alpha)[\mathbb{E}(X)-1](\frac{1}{d}-1)E_0$$

[15] If the bank can choose whether to disclose private information, the bank does not have any incentive to disclose private information to outside investors. This argument, based on an endogenous information structure, will be presented in Section 4.2 in detail.

[16] This leads to a downward jump in social welfare, which will be discussed in Section 5 in detail.

$$+q(1-p)\{(1-\beta_a)[\mathbb{E}(Y)-1](\frac{1}{d}-1)X_L E_0\}. \tag{17}$$

Social welfare, of course, diminishes when (8) is not satisfied:

$$[\mathbb{E}(X)-1]\frac{E_0}{d}+q(1-p)\{[\mathbb{E}(Y)-1]\frac{X_L E_0}{d}\}. \tag{18}$$

Without any regulation, the distribution of assets and bargaining power determine whether (8) holds unless $\underline{d}>d^*$. If the government stipulates mandatory skin in the game $\overline{d}>d^*$, it induces lower level of social welfareby incentivizing the bank to exploit profitable investment projects partially. This is one of our main arguments, and will be discussed in detail with numerical examples and figures in Section 5.

4.2 Endogenously Determined Information Structure

So far, we have illustrated assuming that the information structure is given exogenously. Hereafter, we discuss the argument presuming that the information structure is endogenously determined, that is, the bank can choose between information symmetry and asymmetry in the initial period, without any costs.[17] That is, the bank will determine to disclose private information if it benefits from the information disclosure in terms of expected returns.

A bank basically prefers information asymmetry to symmetry for $\beta_a>\beta_s$ unless the costs of information asymmetry — i.e., the depreciation of the asset price — are too high. There exist two cases under information asymmetry, as discussed in the previous subsection. One is the case in which the bank utilizes the new projects regardless of the payoff realized from I_X projects, and another is the case in which it exploits them only if low payoff, X_L, is realized. The criterion for which case is adopted among them is (8). If it holds, i.e., $d\le d^*$, the former will be adopted, and if not, the latter will be adopted. We denote A_1 and A_0 for the former and the latter, respectively, and A for the adopted one. The bank, however, cannot affect the determination of A since all parameters in (8) are exogenous.

After A is determined by (8), the bank can choose to disclose private information if the expected profit under symmetric information is higher than that under asymmetric information. That is, the bank can choose between S (the case under symmetric information) and A to maximize its expected return at the initial period. If A_1 is adopted, i.e., $d\le d^*$, the bank compares (2) with (13) and decides not to disclose private information if (13) is larger than (2). It can be arranged with respect to d as follows:[18]

$$d\le\frac{\beta_a P^*-\beta_s\mathbb{E}(X)}{\frac{T_1}{q[\mathbb{E}(Y)-1]}+(1-\beta_s)\mathbb{E}(X)-(1-\beta_a)P^*}=:d_1. \tag{19}$$

If A_0 is adopted by (8), i.e., $d>d^*$, the bank compares (2) with (16) and chooses to stay under asymmetric information if (16) is larger than (2). It is equivalent to the following:[19]

[17] If the bank decides this in the interim period, the bank will disclose the private information only when X_H is realized, which leads to the same situation regarding symmetric information,because investors recognize the non-disclosure of private information as a sign of the low payoff of X_L. To avoid this, we postulate that the bank will decide to disclose private information in the initial period.

[18] Not like d^* defined by (8), the denominator of d_1 in (19) can be negative. If it is negative, a sign of inequality in (19) has to be substituted with the opposite one.

[19] Likewise, the denominator of d_0 in (20) can be negative depending on parameters. If this is the case, the sign of inequality has to be substituted with the opposite one.

$$d \leq \frac{(1-p)\beta_a X_L - \beta_s \mathbb{E}(X)}{(1-\beta_s)\mathbb{E}(X) - (1-p)(1-\beta_a)X_L} =: d_0. \tag{20}$$

These arguments can be summarized by the following flowchart (see Figure 1):[20]

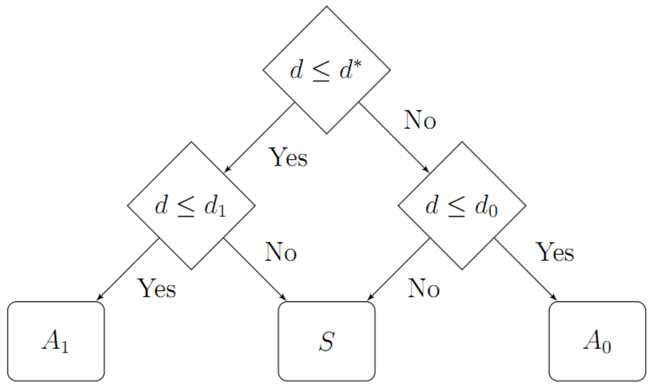

Figure 1. Flowchart for behavior of the bank without leverage

The analysis regarding the expected profits of the bank, investors, and social welfare when information structure is exogenously given is provided in the previous subsection. When the structure of information is endogenous, the analysis depends on the level of skin in the game. At this point investigating the efficiency of regulation regarding skin in the game becomes a necessity. Welfare analysis regarding the level of regulation is presented in the following section.

5. Welfare Analysis

In this section, we analyze how the bank's behavior to maximize its expected return affects the structure of social welfare. To put it briefly, there might exist jumps in the social welfare, either downward or upward, or even both, as the skin in the game increases.

5.1 Structure of Social Welfare

In Section 4.2, we illustrated how the information structure is determined to maximize the bank's expected return. If the bank behaves following Figure 1, there might exist jumps in social welfare as the skin in the game varies because the bank might not undertake profitable investment projects, and the amount of capital invested might decrease due to information asymmetry after skin in the game passes a threshold. We categorize a number of the structure of social welfare and examine the conditions for each type to hold. It is straightforward to attain the conditions based on Figure 1, and thus, we omit the proof of them.[21]

First, there might not exist any jump in social welfare. That is, the bank always prefers information symmetry to asymmetry if a certain condition is satisfied, and we call this the S structure. The condition for this structure to be adopted is as follows:

$$\begin{cases} \underline{d} > d_0 & \text{if} \quad \underline{d} > d^*, \\ \underline{d} > \max(d_1, d_0) & \text{if} \quad \underline{d} \leq d^*. \end{cases} \tag{21}$$

[20] For simplicity, we only consider the case in which the denominators of both d_1 and d_0 are positive in Figure 1.

[21] We illustrate the conditions assuming that the denominators of d_1 and d_0 are positive, because the analysis hereafteris based on Figure 1. We can presume these conditions without a loss of generality because the structure of social welfare does not change even if the denominators are negative and the conditions differ from those presented in this subsection.

The following (Figure 2) is a numerical example of the S structure.

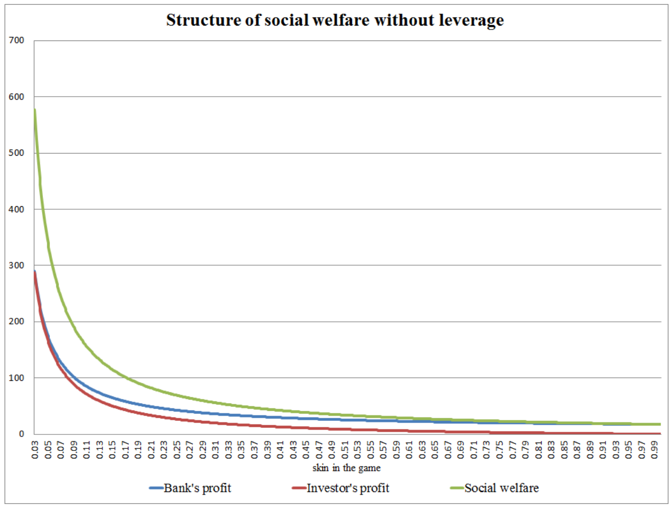

Figure 2. **S** structure
$(X_H = 1.5,\ X_L = 0.8,\ p = 0.5,\ \mathbb{E}(Y) = 1.05,\ q = 0.4,\ \alpha = 0.5,\ \beta_s = 0.4,\ \beta_a = 0.6)$

For the A_1 structure where only A_1 is adopted throughout the entire range of skin in the game, the following condition is required:

$$1 \leq \min\,(d^*, d_1). \tag{22}$$

If this is the case, the bank always fully utilizes the new projects, regardless of skin in the game. The condition, however, is not satisfied under reasonable conditions as (22) is equivalent to the following, which requires the expected return of the new projects to be extremely high:

$$\mathbb{E}(Y) - 1 \geq \frac{X_H - P^*}{P^*}$$
$$\text{and} \tag{23}$$
$$\mathbb{E}(Y) - 1 \geq \frac{\{1 - p(1-q)\}(1-\alpha)[\mathbb{E}(X) - 1] + p(1-q)[\mathbb{E}(X) - X_L]}{\beta_a P^* - \beta_s \mathbb{E}(X)}.$$

A condition the A_0 structure, in which the bank partially utilizes the new projects throughout the entire range of skin in the game is as follows:

$$\underline{d} > d^* \qquad \text{and} \qquad 1 < d_0. \tag{24}$$

This, however, also never happens with reasonable parameters as (24) is equivalent to the following:

$$1 > \frac{\beta_a[\mathbb{E}(Y) - 1]P^*}{\underline{d}\{X_H - P^* - (1-\beta_a)[\mathbb{E}(Y) - 1]P^*\}} \qquad \text{and} \qquad \mathbb{E}(X) < (1-p)X_L. \tag{25}$$

Now we consider the case in which one jump occurs in social welfare. If the jump occurs only once, it is always upward one as social welfare under symmetric information always dominates that under asymmetric information. There are two of this type: the A_1-S structure and the A_0-S structure. In the former, the bank fully invests in the new projects exploiting private information when skin in the game is low and discloses private information after d passes a threshold, d_1. A condition for the A_1-S structure to be adopted is as follows:

$$\underline{d} < \min\,(d^*, d_1) \qquad \text{and} \qquad d^* > \max\,(\underline{d}, d_0). \tag{26}$$

The following (Figure 3) is a numerical example of the A_1-S structure:

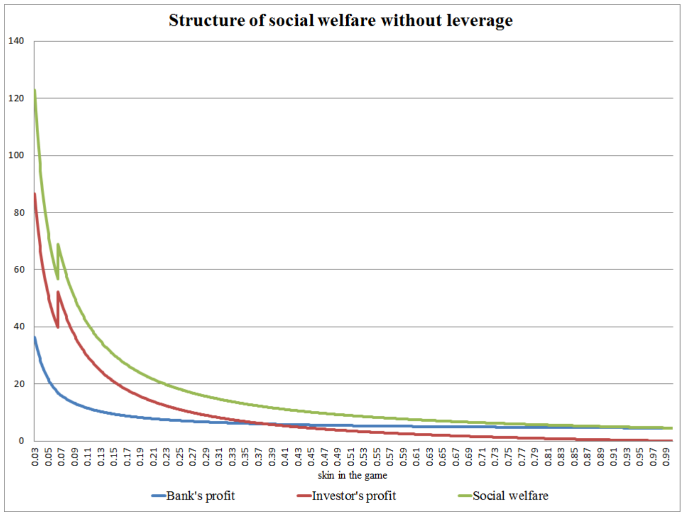

Figure 3. A_1- S structure

$(X_H = 1.5,\ X_L = 0.55,\ p = 0.48,\ \mathbb{E}(Y) = 1.2,\ q = 0.1,\ \alpha = 0.1,\ \beta_s = 0.3,\ \beta_a = 0.7)$

Another structure with one jump in social welfare, the A_0-S structure, implies that the bank benefits from private information and utilizes investment opportunities partially when d is low, but prefers information symmetry to asymmetry after d passes a threshold, d_0. For this structure to hold, the following condition is necessary:

$$\begin{cases} \underline{d} < d_0 & \text{if} \quad \underline{d} > d^*, \\ d_1 < \underline{d} < d_0 & \text{if} \quad \underline{d} \leq d^*. \end{cases} \tag{27}$$

The following (Figure 4) is a numerical example of the A_0-S structure:

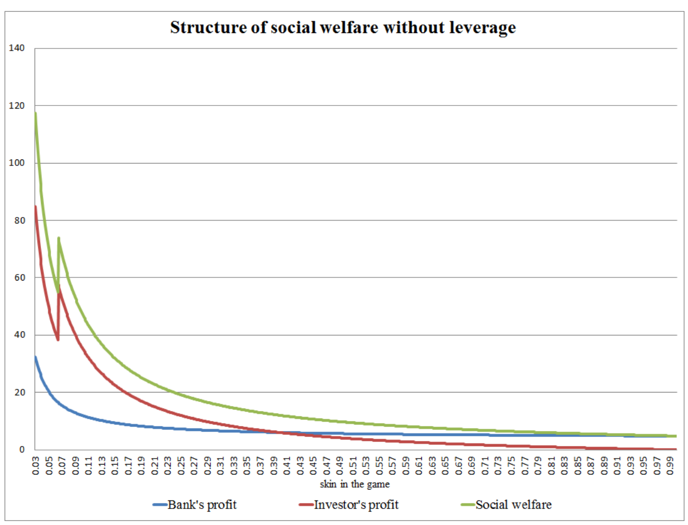

Figure 4. **A_0-S** structure

$(X_H = 1.55,\ X_L = 0.4,\ p = 0.55,\ \mathbb{E}(Y) = 1.05,\ q = 0.3,\ \alpha = 0.2,\ \beta_s = 0.1,\ \beta_a = 0.9)$

There might exist two jumps in social welfare, both downward one and upward one, under certain circumstances. This A_1-A_0-S structure is one of the most intriguing results of our research. This structure implies that the bank undertakes the new projects whenever it is available under asymmetric information if d is low enough but does not utilize them unless X_L is realized after d exceeds d^*, which yields a downward jump in social welfare.[22] The bank prefers information symmetry to asymmetry if d passes a higher threshold, d_0, and this contributes to the upward jump in social welfare. The condition for this interesting structure to be adopted is as follows, and it holds with reasonable parameters:

$$\underline{d} \leq d^* < \min(d_1, d_0). \tag{28}$$

The following (Figure 5) is a numerical example of the structure with two jumps.

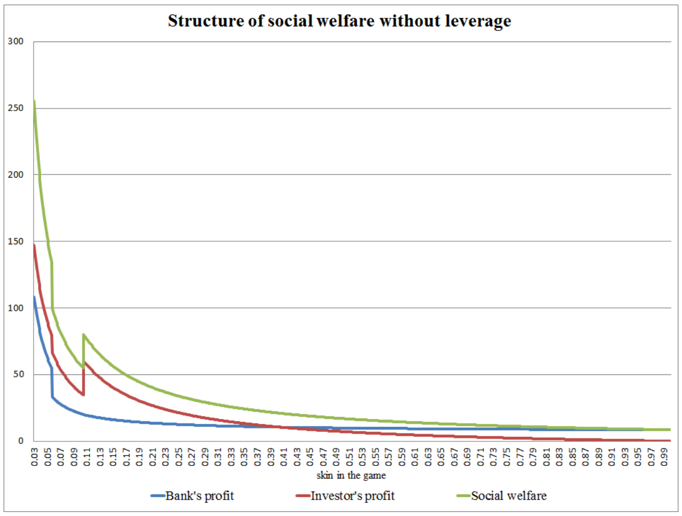

Figure 5. $\boldsymbol{A_1}$-$\boldsymbol{A_0}$-$\boldsymbol{S}$ structure

$(X_H = 1.5,\ X_L = 0.5,\ p = 0.55,\ \mathbb{E}(Y) = 1.055,\ q = 0.6,\ \alpha = 0.2,\ \beta_s = 0.1,\ \beta_a = 0.9)$

There might exist a structure with three jumps in social welfare, the A_1-S-A_0-S structure if the following condition is satisfied, but it never happens with reasonable parameters:

$$\underline{d} < d_1 < d^* < d_0. \tag{29}$$

5.2 Implications for Regulation

This research started with the question that whether the current regulations concerning risk retention are sufficient for incentivizing financial intermediaries and revitalize securitization markets. In this subsection, we clarify the possible side effects of the current regulation and examine comparative statics that support our arguments.

5.2.1 *Possible Side Effects*

The bank adopts strategies that maximize its expected return. It is apparent that securitization augments a bank's expected return tremendously, so the bank lowers the level of d as much as possible and chooses $\underline{d}$, which represents the lower bound of skin in the game, unless there exists regulation stipulated by the authorities or other huge losses caused by severe principal-agent problems.

[22] This jump essentially arises from $(1-p)$ multiplied to the terms in (16).

If the government requires financial institutions to retain a fixed portion of assets they issue without considering features of individual assets or macroeconomic condition, it might entail losses in social welfare — a downward jump in social welfare. $\overline{d}$, which stands for the level of regulation, might come after the downward jump in social welfare. In other words, $\overline{d}$ might be located in the A_0 interval of the A_1-A_0-S structure. This is a huge loss in social welfare in the sense that the bank does not utilize profitable investment opportunities fully due to the regulation stipulated to stabilize the markets. We can further clarify social losses by the following graph (Figure 6) which presents the difference in a bank's profit, investor's profit, and social welfare under the adopted structure and those under symmetric information.

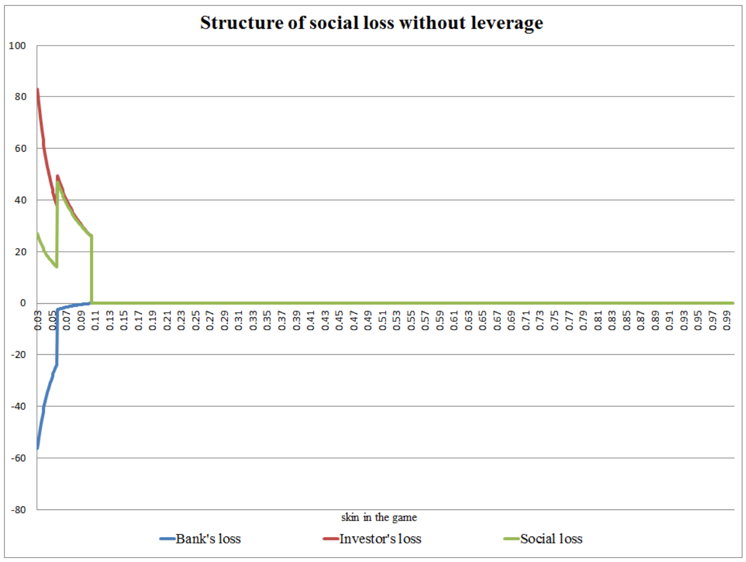

Figure 6. $\boldsymbol{A_1}$**-**$\boldsymbol{A_0}$**-**$\boldsymbol{S}$ structure

$(X_H = 1.5,\ X_L = 0.5,\ p = 0.55,\ \mathbb{E}(Y) = 1.055,\ q = 0.6,\ \alpha = 0.2,\ \beta_s = 0.1,\ \beta_a = 0.9)$

We can see that the bank's loss is always non-positive throughout the entire range of skin in the game, which implies that the bank always benefits from information asymmetry. It can also be checked that the government's policy regarding risk retention does not resolve the problem caused by misalignment of maximization of bank's profit and that of social welfare unless it is extremely tight.

If the structure with one upward jump is adopted, $\overline{d}$ might come right before the upward jump of social welfare occurs. This implies that the regulation is not enough to incentivize financial institutions to disclose private information, and this result can be found in the A_1-S and A_0-S structure. Figure 3 and Figure 4 correspond to this case. The following (Figure 7) is a graph that shows social losses with respect to the level of skin in the game:

The level of regulation might be slightly lower than a threshold that makes the bank disclose private information. In that case, tightening regulation a little more might enhance social welfare substantially.

There any jump might not exist in social welfare throughout the entire range of skin in the game. The S structure corresponds to this case, and the regulation of risk retention does not affect the incentives of the bank. It rather lowers social welfare as long as $\overline{d} > \underline{d}$.

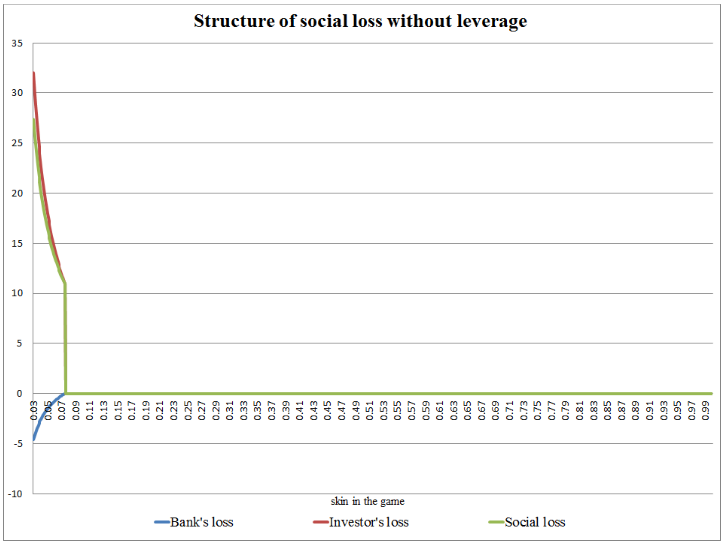

Figure 7. $\boldsymbol{A_1}$-$\boldsymbol{S}$ structure

($X_H = 1.5,\ X_L = 0.55,\ p = 0.48,\ \mathbb{E}(Y) = 1.2,\ q = 0.1,\ \alpha = 0.1,\ \beta_s = 0.3,\ \beta_a = 0.7$)

5.2.2 *Comparative Statics*

Now we examine comparative statics of essential figures in our analysis and their implications for regulation. One of the kernels in our analysis is d^*, a threshold of downward jump in social welfare. We can verify from the following that d^* increases as $\mathbb{E}(Y)$ and q increase:

$$\frac{\partial d^*}{\partial \mathbb{E}(Y)} = \frac{\beta_a P^*}{X_H - P^* - (1-\beta_a)[\mathbb{E}(Y)-1]P^*} \times \left(1 + \frac{(1-\beta_a)[\mathbb{E}(Y)-1]P^*}{X_H - P^* - (1-\beta_a)[\mathbb{E}(Y)-1]P^*}\right) > 0, \tag{30}$$

$$\frac{\partial d^*}{\partial q} = \frac{\beta_a[\mathbb{E}(Y)-1]}{X_H - P^* - (1-\beta_a)[\mathbb{E}(Y)-1]P^*} \times \left(\overline{P} - X_L - \frac{(X_L - \overline{P} - (1-\beta_a)[\mathbb{E}(Y)-1](\overline{P} - X_L))P^*}{X_H - P^* - (1-\beta_a)[\mathbb{E}(Y)-1]P^*}\right) > 0. \tag{31}$$

In other words, the bank is more likely to utilize the new projects fully in a boom, and less likely to invest in them during recession. This implies that the strict regulation of risk retention in recession — i.e., lowering $\overline{d}$ when $\mathbb{E}(Y)$ and q are low — might aggravate the market of securitization, and thus, countercyclical regulation has to be adopted to revitalize the market. The necessity of countercyclical financial regulation is valid and acceptable today. One of the main principles of Basel III is countercyclical capital requirement, and this idea is consistent with recent papers. For instance, Kashyap, Rajan, and Stein (2008) alleged that the regulation needs to be time-variant. They adopt an analogy that stresses the necessity of time-variant regulation as follows: "Time-variant capital requirements are analogous to forcing a homeowner to hold a fixed fraction of his house's value in savings, as a hedge against storm damage, and then not letting him spend down these savings when a storm hits. Given this restriction, the homeowner will have no choice but to sell the damaged house and move to a smaller place, i.e., to suffer an economic contraction." There exist a number of other studies that are in line with this argument (e.g. Brunnermeier, Crocket, Goodhart, Persaud, & Shin, 2009; BIS, 2010; Adrian & Brunnermeier, 2011).

We have analyzed regarding the threshold of the jumps in social welfare. Now we examine the depth of the jumps in social welfare. In an A_1-A_0-S structure, there exist two jumps in social welfare: downward one and upward one. We denote the depth of a downward jump and an upward jump as J_d and J_u, respectively. J_d is the difference between (15) and (18) with skin in the game d^*, and can be written as follows:

$$J_d = \frac{q[\mathbb{E}(Y)-1][P^*-(1-p)X_L]E_0}{d^*}]$$

$$= \frac{q[P^*-(1-p)X_L]\{X_H - P^* - (1-\beta_a)[\mathbb{E}(Y)-1]P^*\}E_0}{\beta_a P^*}. \tag{32}$$

Comparative statics with respect to the depth of the downward jump are as follows:

$$\frac{\partial J_d}{\partial \mathbb{E}(Y)} = -\frac{q(1-\beta_a)[P^*-(1-p)X_L]E_0}{\beta_a} < 0, \tag{33}$$

$$\frac{\partial J_d}{\partial q} = \frac{q[\mathbb{E}(Y)-1]E_0(\overline{P}-X_L)[P^*-(1-p)X_L]}{d^*}$$

$$\times \left(\frac{1}{P^*-(1-p)X_L} - \frac{1}{P^*} + \frac{1}{q(\overline{P}-X_L)} - \frac{1+(1-\beta_a)[\mathbb{E}(Y)-1]}{X_H - P^* - (1-\beta_a)[\mathbb{E}(Y)-1]P^*}\right). \tag{34}$$

(33) shows that the depth of a downward jump increases when the expected return of the new projects is low. This result implies that social welfare is more likely to drop, and loss from the drop worsens during a recession. The sign of (34) can be either positive or negative, depending on parameters.

J_u, the depth of the upward jump in social welfare, is the difference between (4) and (18) with skin in the game d_0, and can be written as follows:

$$J_u = \frac{q[\mathbb{E}(Y)-1]\{\mathbb{E}(X)-(1-p)X_L\}E_0}{d_0} = \frac{q[\mathbb{E}(Y)-1]X_H E_0}{d_0}. \tag{35}$$

Comparative statics with respect to the depth of the upward jump are as follows:

$$\frac{\partial J_u}{\partial \mathbb{E}(Y)} = \frac{q X_H E_0}{d_0} > 0, \tag{36}$$

$$\frac{\partial J_u}{\partial q} = \frac{[\mathbb{E}(Y)-1]X_H E_0}{d_0} > 0. \tag{37}$$

These results imply that the depth of an upward jump is small when the expected return and availability of the new projects are low — that is, during a recession — in contrast with that of a downward jump which is large during a recession.

Now we analyze comparative statics of the width of interval A_0 in which the bank utilize the new projects partially. If we denote it as I, it can be represented as follows:

$$I = d_0 - d^*. \tag{38}$$

d_0, however, does not depend on parameters related to the new projects, and thus, comparative statics of the width of interval A_0 are as follows:

$$\frac{\partial I}{\partial \mathbb{E}(Y)} = -\frac{\partial d^*}{\partial \mathbb{E}(Y)} < 0, \tag{39}$$

$$\frac{\partial I}{\partial q} = -\frac{\partial d^*}{\partial q} < 0. \tag{40}$$

These outcomes imply that the interval in which the bank is less likely to utilize the new investment opportunities gets wider during a recession. Hence, we can assert that countercyclical regulation is necessary considering not only the thresholds of the jumps but also the depth of them and the interval in which the new projects are not utilized fully.

We can summarize that during a recession, the threshold of the skin in the game regarding the downward jump decreases, and the downward jump depth increases while that of upward jump decreases. From this analysis, we can infer that tight regulation of risk retention with a fixed ratio during a recession might result in the decrease of social welfare, and thus, countercyclical regulation has to be considered to incentivize the banks in the right way without harming social welfare.

6. Extension to Levered Bank

This section augments the robustness of our model. We show that the argument that we have discussed still holds true, even if we introduce liabilities borrowed from outside investors in the initial period. Furthermore, we can explain what Shleifer and Vishny (2010) addressed, the volatile financial market, even without any assumption of irrationality.

6.1 Model Setup

The basic setup, e.g., the process of financing projects and distributing them and the regulation of risk retention, is same as before. We only illustrate the additional setup here. The bank can borrow from outside investors using securities on its balance sheet as collateral to utilize more projects. The borrowing contract between the bank and investors takes the form of short-term debt, and it requires the bank to maintain a haircut level $h \in (0,1]$ in every period.[23]

That is, if market value of collateral falls, the bank has to liquidate a portion of assets it is holding for a haircut, and this works exactly the same way as regulatory capital requirements. If we denote L_t and E_t the amount of liabilities and equity capital at time t, the following has to be satisfied.

$$\frac{E_t}{E_t + L_t} = h, \qquad \forall t \in \{0,1,2\}. \tag{41}$$

Because this also has to hold at the initial period, it is straightforward that L_0 is determined as follows:

$$L_0 = \frac{1-h}{h} E_0. \tag{42}$$

Here, we impose a restriction to preclude any possibility of default, which is not a main topic in this paper. If the following condition holds, default of the bank never occurs:[24]

$$X_L \geq 1 - h. \tag{43}$$

[23] $h = 1$ corresponds to the situation in which the bank does not have liabilities.

[24] A detailed explanation of the no default condition is presented in Appendix B.

6.2 Exogenously Fixed Information Structure

6.2.1 *Symmetric Information*

If $P_1 < 1$ — that is, if X_L is realized at interim period — the bank has to liquidate a portion of assets on its balance sheet to keep a haircut — i.e., to satisfy the condition (5). If we denote S the amount of assets liquidated for a haircut, it is evaluated as follows by straightforward calculation:[25]

$$S = \frac{E_0}{h}\left(\frac{1-h}{h}\frac{1-P_1}{P_1}\right). \tag{44}$$

Under symmetric information, either $P_1 = X_H$ or $P_1 = X_L$ is realized, and the latter is the only case in which mandatory liquidation is enforced. For $P_1 = X_L$, we denote S_L the corresponding amount of liquidation.[26]

If the bank has liabilities, the 'no cash hoarding condition' specified in Appendix A might not hold depending on parameters even under the symmetric information. That is, it is possible that the bank does not utilize I_X projects that have a positive net present value, and hoards all of its capital as cash. This is because the amount of capital available for the new investment at time 1 decreases due to the mandatory liquidation. For tractability, we only deal with the situation in which no hoarding condition holds hereafter.

Because of the mandatory liquidation, expected profit of the bank and investors at the initial period, and thus the social welfare, are not just $1/h$ times that without leverage. The amount of capital that can be invested in the new projects when X_L is realized is $X_L(E_0/h - S_L)$, not just $X_L E_0/h$, because of the liquidation carried out for a haircut. If X_H is realized with probability p, however, the bank does not have to liquidate any assets. Thus, $X_H E_0/h$ can be used to utilize the new projects if X_H is realized.

Based on these arguments, we can write the bank's expected profit from I_Y projects as follows:

$$\begin{aligned}
&q[p\{\beta_s[\mathbb{E}(Y)-1]\frac{X_H E_0}{dh} + (1-\beta_s)[\mathbb{E}(Y)-1]\frac{X_H E_0}{h}\} \\
&\quad +(1-p)\{\beta_s[\mathbb{E}(Y)-1]\frac{X_L(\frac{E_0}{h}-S_L)}{d} + (1-\beta_s)[\mathbb{E}(Y)-1]X_L(\frac{E_0}{h}-S_L)\}] \\
&= q[\beta_s[\mathbb{E}(Y)-1]\frac{\mathbb{E}(X)E_0}{dh} + (1-\beta_s)\frac{\mathbb{E}(X)E_0}{h} \\
&\quad -(1-p)\{\beta_s[\mathbb{E}(Y)-1]\frac{X_L S_L}{d} + (1-\beta_s)[\mathbb{E}(Y)-1]X_L S_L\}].
\end{aligned} \tag{45}$$

The terms in the last row of (45), $q(1-p)\{\beta_s[\mathbb{E}(Y)-1]\frac{X_L S_L}{d} + (1-\beta_s)[\mathbb{E}(Y)-1]X_L S_L\}$, are a loss incurred by mandatory liquidation for a haircut, which can be interpreted as the market participants' fear of illiquidity.

Thus, the bank's expected profit at the initial period is as follows:

$$\begin{aligned}
&\alpha[\mathbb{E}(X)-1]\frac{E_0}{dh} + (1-\alpha)[\mathbb{E}(X)-1]\frac{E_0}{h} \\
&\quad +q[\beta_s[\mathbb{E}(Y)-1]\frac{\mathbb{E}(X)E_0}{dh} + (1-\beta_s)[\mathbb{E}(Y)-1]\frac{\mathbb{E}(X)E_0}{h}
\end{aligned} \tag{46}$$

[25] Refer to p.313 of Shleifer and Vishny (2010) for a detailed explanation.

[26] P_1 might not be X_L even though X_L is realized under asymmetric information, as we have seen in the argument without liabilities. Thus, the amount of required liquidation also differs from S_L when the information is asymmetric, and this will be discussed afterward.

$$-(1-p)\{\beta_s[\mathbb{E}(Y)-1]\frac{X_L S_L}{d}+(1-\beta_s)[\mathbb{E}(Y)-1]X_L S_L\}].$$

Expected profit of investors can be evaluated in the same way:

$$\begin{aligned}&(1-\alpha)[\mathbb{E}(X)-1](\frac{1}{d}-1)\frac{E_0}{h}\\&+q[(1-\beta_s)[\mathbb{E}(Y)-1](\frac{1}{d}-1)\frac{\mathbb{E}(X)E_0}{h}\\&-(1-p)\{(1-\beta_s)[\mathbb{E}(Y)-1](\frac{1}{d}-1)X_L S_L\}].\end{aligned} \tag{47}$$

Social welfare, the sum of (46) and (47), is as follows:

$$[\mathbb{E}(X)-1]\frac{E_0}{dh}+q[[\mathbb{E}(Y)-1]\frac{\mathbb{E}(X)E_0}{dh}-(1-p)\{[\mathbb{E}(Y)-1]\frac{X_L S_L}{d}\}]. \tag{48}$$

We can see that a loss of $q(1-p)\{[\mathbb{E}(Y)-1]X_L S_L/d\}$ occurs in social welfare due to a haircut.

6.2.2 *Asymmetric Information*

The argument regarding asymmetric information is more complicated when the bank has liabilities. The problem is that P^* can be below 1 even though (8) is satisfied, which means that the bank has to liquidate a portion of assets for a haircut, regardless of the payoff realized from I_X projects. This is a loss of social welfare incurred by information asymmetry and creditors' fear of illiquidity.

First of all, suppose that (8) holds, and $P^* \geq 1$. In that case, the bank does not have to liquidate any assets, and thus, it can fully utilize the new projects. Transfer of a portion of an expected profit from the bank to investors occurs. Expected profit of the bank at time 0 is as follows:

$$\begin{aligned}&\alpha[\mathbb{E}(X)-1]\frac{E_0}{dh}+(1-\alpha)[\mathbb{E}(X)-1]\frac{E_0}{h}-\frac{T_1 E_0}{h}\\&+q\{\beta_a[\mathbb{E}(Y)-1]\frac{P^* E_0}{dh}+(1-\beta_a)[\mathbb{E}(Y)-1]\frac{P^* E_0}{h}\}.\end{aligned} \tag{49}$$

This is exactly $1/h$ times (13) which is the case where there is no leverage and (8) is satisfied. Expected profit of investors is also $1/h$ times of (13) as follows:

$$\begin{aligned}&(1-\alpha)[\mathbb{E}(X)-1](\frac{1}{d}-1)\frac{E_0}{h}+T_1\frac{E_0}{h}\\&+q\{(1-\beta_a)[\mathbb{E}(Y)-1](\frac{1}{d}-1)\frac{P^* E_0}{h}\}.\end{aligned} \tag{50}$$

Social welfare, the sum of (49) and (50), is as follows:

$$[\mathbb{E}(X)-1]\frac{E_0}{dh}+q\{[\mathbb{E}(Y)-1]\frac{P^* E_0}{dh}\}. \tag{51}$$

As we can see above, leverage amplifies social welfare $1/h$ times without causing any adverse effect as long as (8) holds and $P^* \geq 1$.

Now we consider the case in which (8) holds but $P^* < 1$. If $P^* < 1$, the bank always has to liquidate the assets on the balance sheet to keep a haircut level. Here, however, we need an additional assumption to proceed with the argument. If investors know that the bank has to liquidate assets on its balance sheet regardless of the realization of X or availability of I_Y projects for

$P^* < 1$, the price of assets at time 1 will rise and converge to $\overline{P} > 1$, which means that the bank does not have to liquidate them. Then the bank would not sell assets if X_H is realized and the I_Y project is not available. This means that the asset price will fall and converge to $P^* < 1$ again. In this manner, the asset price keeps hovering between $P^* < 1$ and $\overline{P} > 1$, not stabilized at an equilibrium price. Here, we assume that $P_1 = P^* < 1$ due to conservative risk assessment in the market, which is a universal phenomenon in the real world.[27]

If we assume $P_1 = P^* < 1$, a stricter condition has to be satisfied regarding skin in the game for the bank to utilize the new projects. This is because the amount of capital that can be invested in new projects decreases from P^*E_0/h to $P^*(E_0/h - S^*)$ due to the mandatory liquidation, where S^* denotes the amount of required liquidation corresponding to $P_1 = P^*(< 1)$. The condition is as follows:

$$(X_H - P^*)\frac{E_0}{h} \leq \beta_a[\mathbb{E}(Y) - 1]\frac{P^*(\frac{E_0}{h} - S^*)}{d} + (1 - \beta_a)[\mathbb{E}(Y) - 1]P^*(\frac{E_0}{h} - S^*), \tag{52}$$

which can be arranged with respect to d as follows:[28]

$$d \leq \frac{\beta_a[\mathbb{E}(Y) - 1]P^*(1 - \frac{1-h}{h}\frac{1-P^*}{P^*})}{X_H - P^* - (1 - \beta_a)[\mathbb{E}(Y) - 1]P^*(1 - \frac{1-h}{h}\frac{1-P^*}{P^*})} =: d^{L*}. \tag{53}$$

If (53) is satisfied, the bank would utilize the new projects after the mandatory liquidation. If not, the bank would abandon profitable investment when X_H is realized even though (8) holds. This induces the same result as when (8) is not satisfied.

Let us illustrate the case in which (53) holds first. The bank's expected return from the assets on the balance sheet is more complicated than the other cases mentioned before. The bank has to liquidate at least S^* amount of the assets for a haircut regardless of the payoff X and availability of I_Y projects. If the new projects are available, the bank would sell the rest of the assets after the mandatory liquidation. The same result will take place when the new projects are not available and X_L is realized. If the new projects are not available and X_H is realized, however, the bank would sell only S^* amount of assets. Based on this argument, the bank's expected return from each asset on the balance sheet by straightforward calculation given in Appendix C is as follows:

$$\begin{aligned} q[(P^* - P_0)\frac{E_0}{h}] + (1-q)[p\{(X_H - P_0)(\frac{E_0}{h} - S^*) + (P^* - P_0)S^*\} + (1-p)\{(P^* - P_0)\frac{E_0}{h}\}] \\ = [P^* - P_0 + p(1-q)(X_H - P^*)]\frac{E_0}{h} - p(1-q)(X_H - P^*)S^*. \end{aligned} \tag{54}$$

Here, we know that the following result holds from (12):

$$[P^* - P_0 + p(1-q)(X_H - P^*)]\frac{E_0}{h} = [(1-\alpha)[\mathbb{E}(X) - 1] - T_1]\frac{E_0}{h}. \tag{55}$$

Regarding the second term in (54), the following holds by straightforward calculation given in Appendix C:

---

[27] One of the basic principles of the financial reform is also a conservative risk assessment. Refer to the U.S. Department of the Treasury (2009).

[28] It is obvious that $d^{L*} < d^*$ always holds.

$$X_H - P^* = q(1-\alpha)[\mathbb{E}(X)-1] + (1-pq)(X_H - X_L). \tag{56}$$

From (55) and (56), we can rewrite (54) as follows:

$$(1-\alpha)[\mathbb{E}(X)-1]\frac{E_0}{h} - \frac{T_1 E_0}{h} - T_2 S^* \tag{57}$$

where

$$T_2 := p(1-q)[q(1-\alpha)[\mathbb{E}(X)-1] + (1-pq)(X_H - X_L)].$$

Expected profit of the bank at the initial period is as follows:

$$\begin{aligned} &\alpha[\mathbb{E}(X)-1]\frac{E_0}{dh} + (1-\alpha)[\mathbb{E}(X)-1]\frac{E_0}{h} - T_1\frac{E_0}{h} - T_2 S^* \\ &+q\{\beta_a[\mathbb{E}(Y)-1]\frac{P^*(\frac{E_0}{h}-S^*)}{d} + (1-\beta_a)[\mathbb{E}(Y)-1]P^*(\frac{E_0}{h}-S^*)\}. \end{aligned} \tag{58}$$

More expected return is transferred from the bank to investors compared to the case in which (8) holds and $P_1 \geq 1$. An additional transfer, $T_2 S^*$, is incurred by a haircut, which can be interpreted as investors' fear of illiquidity. Disposable capital to I_Y projects drops from $P^* E_0/h$ to $P^*(E_0/h - S^*)$ compared to (49), which is a loss of social welfare.

Investors' expected profit at the initial period is as follows:

$$\begin{aligned} &(1-\alpha)[\mathbb{E}(X)-1]\left(\frac{1}{d}-1\right)\frac{E_0}{h} + T_1\frac{E_0}{h} + T_2 S^* \\ &+q\{(1-\beta_a)[\mathbb{E}(Y)-1](\frac{1}{d}-1)P^*(\frac{E_0}{h}-S^*)\}. \end{aligned} \tag{59}$$

Social welfare, the sum of (58) and (59), of course, shrinks compared to (51):

$$[\mathbb{E}(X)-1]\frac{E_0}{dh} + q\{[\mathbb{E}(Y)-1]\frac{P^*(\frac{E_0}{h}-S^*)}{d}\}. \tag{60}$$

If (8) does not hold, or (53) does not hold even though (8) holds, the bank would undertake the new projects only if X_L is realized, and this leads to $P_1 = X_L < 1$. If this is the case, the bank would not sell more than S_L when X_H is realized whether I_Y projects are available. Therefore, the bank's expected return from the assets on the balance sheet is as follows:

$$\begin{aligned} &p\left\{(X_H - P_0)\left(\frac{E_0}{h} - S_L\right) + (X_L - P_0)S_L\right\} + (1-p)(X_L - P_0)\frac{E_0}{h} \\ &= p\left\{(X_H - \overline{P})\frac{E_0}{h} - (X_H - \overline{P})S_L + (X_L - \overline{P})S_L\right\} + (1-p)(X_L - \overline{P})\frac{E_0}{h} \\ &= [\mathbb{E}(X) - \overline{P}]\frac{E_0}{h} - p(X_H - X_L)S_L = (1-\alpha)[\mathbb{E}(X)-1]\frac{E_0}{h} - T_3 S_L \end{aligned} \tag{61}$$

where

$$T_3 := p(X_H - X_L).$$

T_3 is much larger than T_1 or T_2, which implies that the bank suffers more severe losses from information asymmetry and a haircut when (8) or (53) does not hold. Moreover, the amount of capital to utilize the new projects decreases sharply to $X_L(E_0/h - S_L)$. Accordingly, the bank's expected profit at the initial period is as follows:

$$\alpha[\mathbb{E}(X)-1]\frac{E_0}{dh}+(1-\alpha)[\mathbb{E}(X)-1]\frac{E_0}{h}-T_3S_L$$
$$+(1-p)q\{\beta_a[\mathbb{E}(Y)-1]\frac{X_L(\frac{E_0}{h}-S_L)}{d}+(1-\beta_a)[\mathbb{E}(Y)-1]X_L(\frac{E_0}{h}-S_L)\}. \tag{62}$$

The investors' expected profit is as follows:

$$(1-\alpha)[\mathbb{E}(X)-1](\frac{1}{d}-1)\frac{E_0}{h}+T_3S_L+(1-p)q\{\beta_a[\mathbb{E}(Y)-1]X_L(\frac{1}{d}-1)(\frac{E_0}{h}-S_L)\}. \tag{63}$$

Thus, social welfare, the sum of (62) and (63), is as follows:

$$[\mathbb{E}(X)-1]\frac{E_0}{dh}+(1-p)q\{[\mathbb{E}(Y)-1]\frac{X_L(\frac{E_0}{h}-S_L)}{d}\}. \tag{64}$$

This is not just $1/\mathrm{h}$ times (18) due to mandatory liquidation enforced to keep a haircut.

6.3 Endogenously Determined Information Structure

The argument for an endogenously determined information structure when the bank has liabilities is basically the same as the one without leverage, but it needs a few more steps to be considered because of the commitment of a haircut. If (8) holds ($d \le d^*$) and $P^* \ge 1$, the bank invests in the new projects regardless of the payoff X without any mandatory liquidation. We denote A^L_{1+} for this case. This, however, is not the case if $P^* < 1$, even though (8) holds. The bank is required to sell a portion of the assets on the balance sheet if $P^* < 1$. After the liquidation, the amount of capital for the new investment decreases, and another criterion, (53), is required to decide whether to utilize the new projects fully. If (53) holds ($d \le d^{L*}$), the bank expends the capital left after the liquidation to the new investment regardless of the payoff X. We denote this case as A^L_{1-}. If (53) does not hold ($d > d^{L*}$), the bank utilizes the new projects only if X_L is realized, which is same as when (8) does not hold. We denote A^L_0 for this case. A refers to the adopted one among A^L_{1+}, A^L_{1-}, and A^L_0 based on the criteria (8) and (53).

After A is determined, the bank chooses whether to stay under asymmetric information or to disclose private information. We denote S^L for the case under symmetric information. If A^L_{1+} is adopted, the bank compares (46) with (49) and decides not to disclose private information if (49) is larger than (46). This can be arranged with respect to d as follows:[29]

$$d \le \frac{\beta_a P^*(1-\frac{1-h}{h}\frac{1-P^*}{P^*})+\beta_s[(1-p)X_L(\frac{1-h}{h}\frac{1-X_L}{X_L})-\mathbb{E}(X)]}{\frac{T_1-T_2(\frac{1-h}{h}\frac{1-P^*}{P^*})}{q[\mathbb{E}(Y)-1]}+(1-\beta_s)[\mathbb{E}(X)-(1-p)X_L(\frac{1-h}{h}c1-X_LX_L)]-(1-\beta_a)P^*(1-\frac{1-h}{h}\frac{1-P^*}{P^*})} \tag{65}$$
$$=: d^L_{1-}.$$

The same argument is applied when A^L_{1-} is adopted. The bank compares (46) with (58) and stays under asymmetric information if (57) is larger than (46). It is equivalent to the following:[30]

$$d \le \frac{\beta_a P^*(1-\frac{1-h}{h}\frac{1-P^*}{P^*})+\beta_s[(1-p)X_L(\frac{1-h}{h}\frac{1-X_L}{X_L})-\mathbb{E}(X)]}{\frac{T_1-T_2(\frac{1-h}{h}\frac{1-P^*}{P^*})}{q[\mathbb{E}(Y)-1]}+(1-\beta_s)[\mathbb{E}(X)-(1-p)X_L(\frac{1-h}{h}c1-X_LX_L)]-(1-\beta_a)P^*(1-\frac{1-h}{h}\frac{1-P^*}{P^*})} \tag{66}$$
$$=: d^L_{1-}.$$

---

[29] It is possible that the denominator of d^L_{1+} is negative. In that case, the sign of inequality in (65) has to be substituted with the opposite one.

[30] The sign of inequality in (66) has to be substituted with the opposite one if the denominator of d^L_{1-} in (66) is negative.

If A_0^L is adopted — if (8) does not hold, or if (53) does not hold even though (8) holds — the bank compares (46) with (62), and prefers A_0^L to S^L if (62) is larger than (46). This is equivalent to the following inequality:[31]

$$d \leq \frac{(1-p)X_L[\beta_a(1-\frac{1-h}{h}\frac{1-X_L}{X_L})+\beta_s(\frac{1-h}{h}\frac{1-X_L}{X_L})]-\beta_s\mathbb{E}(X)}{\frac{T_3(\frac{1-h}{h}\frac{1-X_L}{X_L})}{q[\mathbb{E}(Y)-1]}+(1-\beta_s)\mathbb{E}(X)-(1-p)X_L[(1-\beta_s)(\frac{1-h}{h}\frac{1-X_L}{X_L})+(1-\beta_a)(1-\frac{1-h}{h}\frac{1-X_L}{X_L})]} \tag{67}$$
$$=: d_0^L.$$

We can sum up the argument as follows (Figure 8):[32]

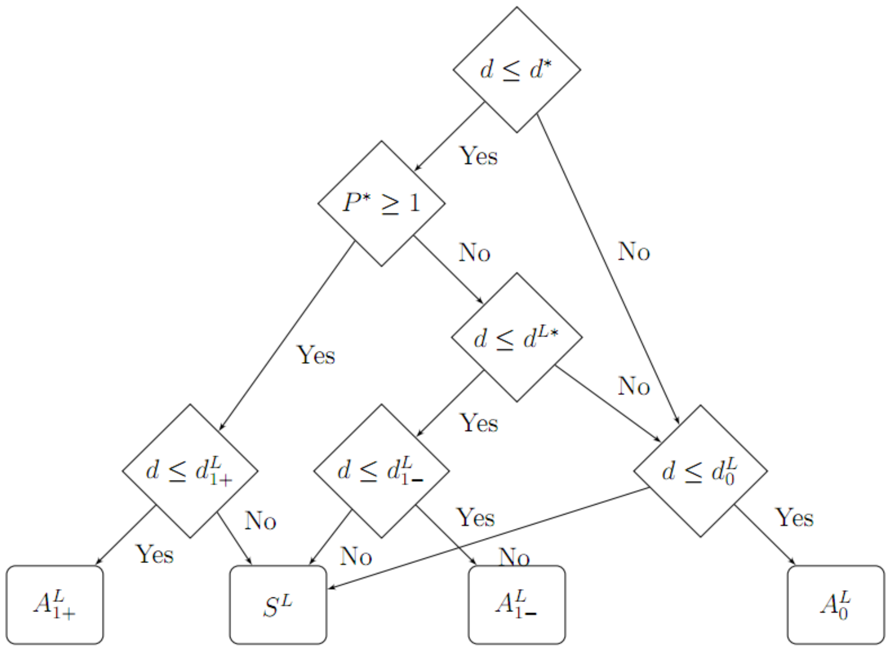

Figure 8. Flowchart for behavior of the bank with leverage

6.4 Welfare Analysis

6.4.1 *Structure of Social Welfare*

Now, we will analyze the structure of social welfare regarding the level of risk retention requirement.[33] The structure with no jump, the S^L structure, is adopted when the following condition is satisfied:

$$\begin{cases} \underline{d} > d_0^L & \text{if} \quad \underline{d} > d^* \\ \underline{d} > \max(d_0^L, d_{1+}^L) & \text{if} \quad \underline{d} \leq d^* \quad \text{and} \quad P^* \geq 1 \\ \underline{d} > d_0^L & \text{if} \quad d^{L*} < \underline{d} \leq d^* \quad \text{and} \quad P^* < 1 \\ \underline{d} > \max(d_0^L, d_{1-}^L) & \text{if} \quad \underline{d} \leq \min(d^*, d^{L*}) \quad \text{and} \quad P^* < 1. \end{cases} \tag{68}$$

The following (Figure 9) is a numerical example of the S^L structure:

[31] If the denominator of d_0^L in (67) is negative, the sign of inequality in (67) has to be substituted with the opposite one.

[32] For simplicity, we only consider the case in which the denominators of d_{1+}^L, d_{1-}^L, and d_0^L are positive in Figure 2.

[33] We illustrate the conditions assuming that the denominators of d_{1+}^L, d_{1-}^L, and d_0^L are positive since the analysis hereafter is based on Figure 8.

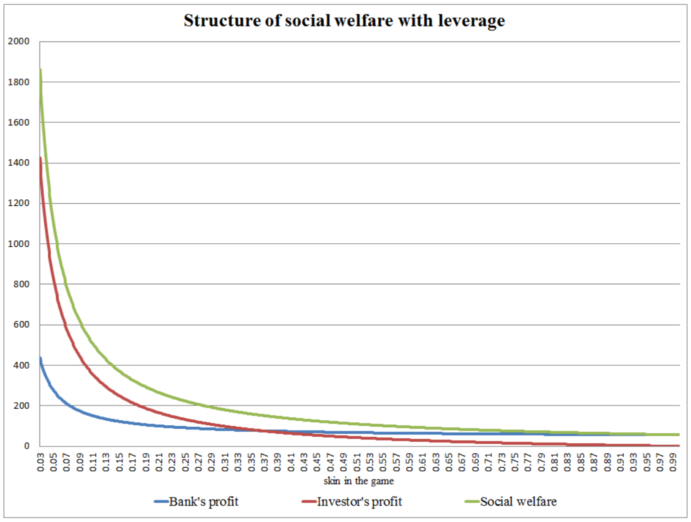

Figure 9. $\boldsymbol{S^L}$ structure

$(X_H = 1.3,\ X_L = 0.8,\ p = 0.6,\ \mathbb{E}(Y) = 1.05,\ q = 0.3,\ \alpha = 0.2,\ \beta_s = 0.3,\ \beta_a = 0.7,\ h = 0.2)$

A condition for the A^L_{1+} structure in which A^L_{1+} is adopted throughout the entire range of skin in the game is as follows:

$$1 \le \min\left(d^*, d^L_1, P^*\right). \tag{69}$$

This structure implies that the bank always invests in new projects, regardless of the level of d and the payoff X without any mandatory liquidation for a haircut. For the liquidation not to occur, either the expected return of the I_X project has to be extremely high, or the ex ante probability that the new project becomes available at the interim period has to be extremely low. Besides, expected return from the new projects has to be extremely high. For these reasons, this structure never appears with moderate parameters.

The A^L_{1-} structure and A^L_0 structure need the following conditions, respectively:

$$P^* < 1 \le \min\left(d^*, d^{L*}, d^L_{1-}\right), \tag{70}$$

$$\underline{d} > d^* \qquad \text{and} \qquad 1 \le d^L_0. \tag{71}$$

Both structures do not show up for similar reasons.

Now we consider the social welfare structure with one jump. Unlike the case without leverage, in which the jump is always upward if it occurs only once, it could be either upward or downward when the bank has liabilities. This is because the mandatory liquidation might not take place under asymmetric information, depending on parameters. Social welfare of A^L_{1+} always dominates that of S^L as the bank does not need to liquidate any assets on the balance sheet, regardless of the payoff X for $P^* \ge 1$. The jump in A^L_{1+}-S^L structure, therefore, is always downward. A^L_{1-}-S^L structure is subtle because A^L_{1-} might dominate S^L or be dominated by S^L in terms of social welfare, depending on the amount of liquidation required, so either is possible in the A^L_{1-}-S^L structure. The jump in the A^L_0-S^L structure, however, is always upward one as social welfare of A^L_0 is always dominated by that of S^L.

The A^L_{1+}-S^L structure is the one in which the bank always utilizes the new projects at the interim period whenever available without any mandatory liquidation while d is low, but chooses to disclose private information if d goes beyond a threshold, d^L_{1+}. This structure holds if the following is satisfied:

$$\underline{d} < \min\left(d^*, d^L_{1+}\right) \qquad \text{and} \qquad \max\left(\underline{d}, d^L_0\right) < d^* \qquad \text{and} \qquad P^* \ge 1. \tag{72}$$

The following (Figure 10) is a numerical example of this A_{1+}^{L}-S^{L} structure:

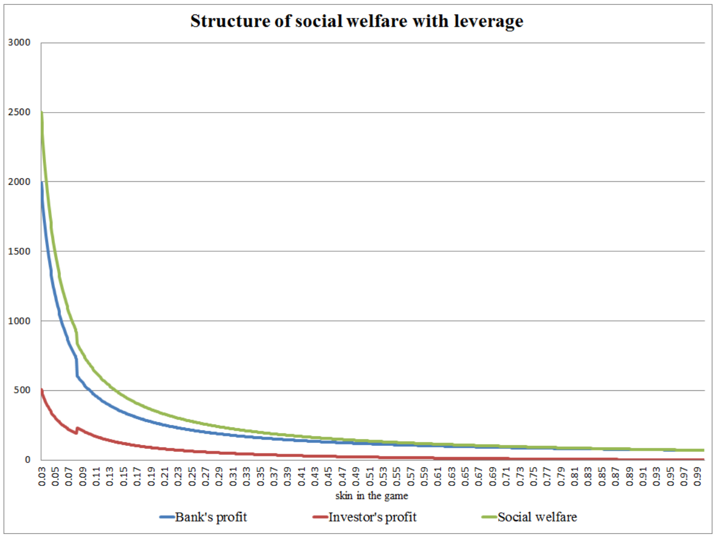

Figure 10. $\boldsymbol{A_{1+}^{L}}$**-**$\boldsymbol{S^{L}}$ structure

$(X_H = 1.5,\ X_L = 0.8,\ p = 0.45,\ \mathbb{E}(Y) = 1.05,\ q = 0.7,\ \alpha = 0.8,\ \beta_s = 0.2,\ \beta_a = 0.8,\ h = 0.2)$

The A_{1-}^{L}-S^{L} structure is similar to A_{1+}^{L}-S structure except that mandatory liquidation is enforced due to $P^* < 1$, and the threshold becomes d_{1-}^{L}. This structure appears when the following condition is satisfied:

$$\underline{d} < \min\left(d^*, d_{1-}^{L}\right) \qquad \text{and} \qquad \max\left(\underline{d}, d_0^{L}\right) < d^* \le d^{L*} \qquad \text{and} \qquad P^* < 1. \tag{73}$$

As mentioned before, either jump is possible. The following (Figure 11) is a numerical example of the A_{1-}^{L}-S^{L} structure with a downward jump:

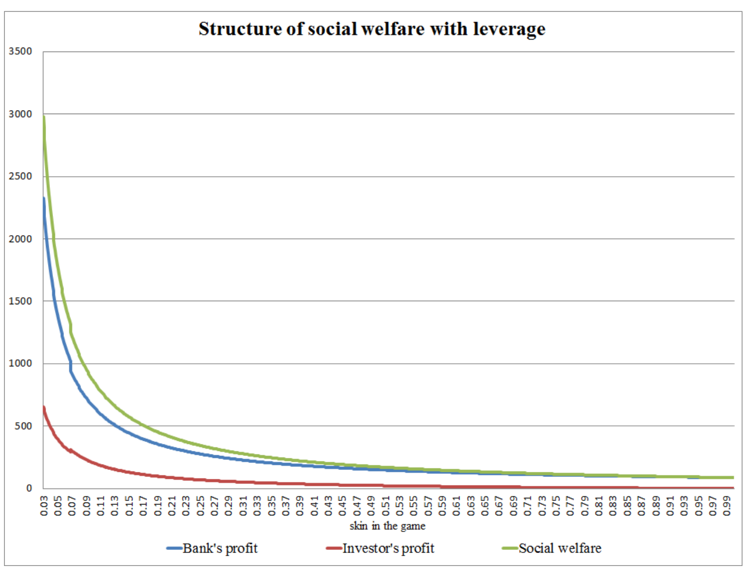

Figure 11. $\boldsymbol{A_{1-}^{L}}$**-**$\boldsymbol{S^{L}}$ structure with downward jump

$(X_H = 1.5,\ X_L = 0.8,\ p = 0.5,\ \mathbb{E}(Y) = 1.05,\ q = 0.6,\ \alpha = 0.8,\ \beta_s = 0.3,\ \beta_a = 0.7,\ h = 0.2)$

The following (Figure 12) is a numerical example of the A_{1-}^{L}-S^{L} structure with an upward jump:

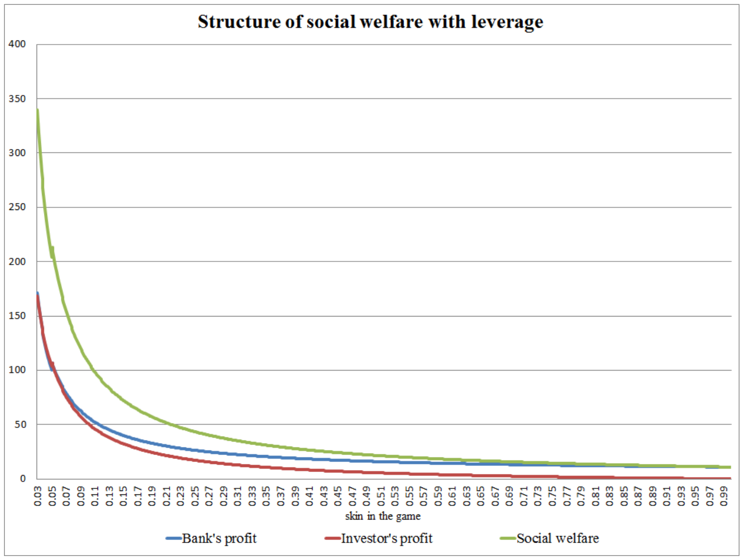

Figure 12. $\boldsymbol{A_{1-}^{L}}$**-**$\boldsymbol{S^{L}}$ structure with upward jump

$(X_H = 1.2,\ X_L = 0.8,\ p = 0.6,\ \mathbb{E}(Y) = 1.05,\ q = 0.3,\ \alpha = 0.5,\ \beta_s = 0.4,\ \beta_a = 0.6,\ h = 0.5)$

The last structure with one jump in social welfare is the A_0^L-S^L structure, which implies that the bank partially utilizes the new projects while d is low and chooses to disclose private information if d passes a threshold, d_0^L. This structure holds if the following condition is satisfied:

$$\begin{cases} \underline{d} < d_0^L & \text{if} \quad \underline{d} > d^* \\ d_{1+}^L < \underline{d} < d_0^L & \text{if} \quad \underline{d} \leq d^* \quad \text{and} \quad P^* \geq 1 \\ \underline{d} < d_0^L & \text{if} \quad d^{L*} < \underline{d} \leq d^* \quad \text{and} \quad P^* < 1 \\ d_{1-}^L < \underline{d} < d_0^L & \text{if} \quad \underline{d} \leq \min\left(d^*, d^{L*}\right) \quad \text{and} \quad P^* < 1. \end{cases} \tag{74}$$

The following (Figure 13) is a numerical example of the A_0^L-S^L structure:

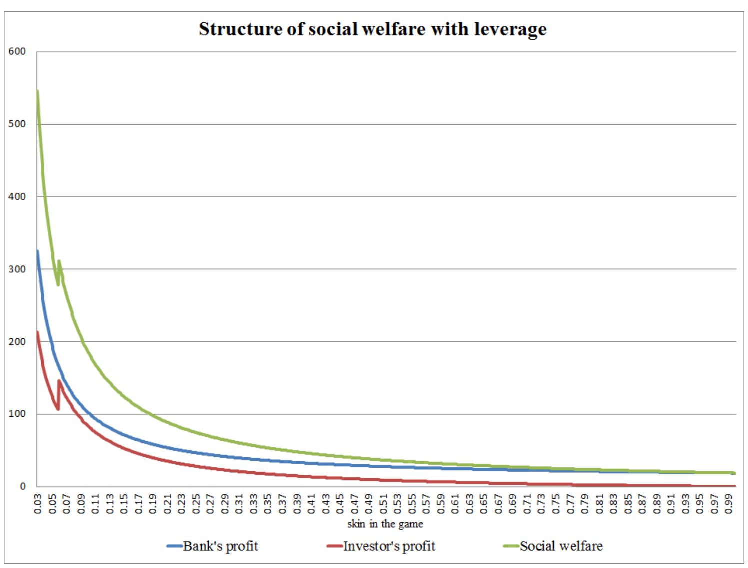

Figure 13. $\boldsymbol{A_0^L}$**-**$\boldsymbol{S^L}$ structure

$(X_H = 1.5,\ X_L = 0.7,\ p = 0.55,\ \mathbb{E}(Y) = 1.05,\ q = 0.5,\ \alpha = 0.6,\ \beta_s = 0,\ \beta_a = 0.35,\ h = 0.9)$

Now we examine the case with two jumps, both upward and downward, in social welfare. This intriguing structure is less likely to be adopted for a levered bank than for an unlevered bank as the mandatory liquidation reduces the amount of capital available to invest in new projects.

In A^L_{1+}-A^L_0-S^L structure, the bank fully utilizes the new projects without any mandatory liquidation at the interim period when d is low. After d passes a threshold, d^*, however, the bank invests to those projects only when the lowpayoff is realized from I_X projects. Exceeding another threshold of skin in the game, d^L_0, the bank rather chooses to disclose private information. This structure appears when the following condition is satisfied:

$$\underline{d} < d^* < \min\left(d^L_{1+}, d^L_0\right) \qquad \text{and} \qquad P^* \geq 1. \tag{75}$$

The following (Figure 14) is a numerical example of the A^L_{1+}-A^L_0-S^L structure:

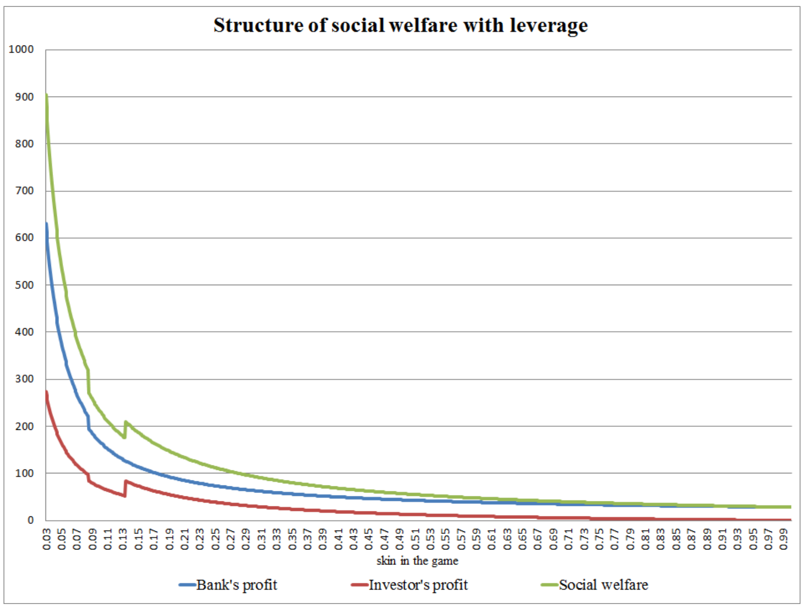

Figure 14. A^L_{1+}-A^L_0-S^L structure

$(X_H = 1.6,\ X_L = 0.7,\ p = 0.55,\ \mathbb{E}(Y) = 1.07,\ q = 0.7,\ \alpha = 0.7,\ \beta_s = 0,\ \beta_a = 0.7,\ h = 0.9)$

Unlike in the case without leverage, there is another structure that includes two jumps for the levered bank: the A^L_{1-}-A^L_0-S^L structure, which is same as in the former case except that the bank undergoes mandatory liquidation for a haircut, and thresholds are d^{L*} and d^L_0. This structure holds when the following condition is satisfied:

$$\underline{d} < d^* < \min\left(d^{L*}, d^L_{1-}, d^L_0\right) \qquad \text{and} \qquad P^* < 1. \tag{76}$$

The following (Figure 15) is a numerical example of the A^L_{1-}-A^L_0-S^L structure:

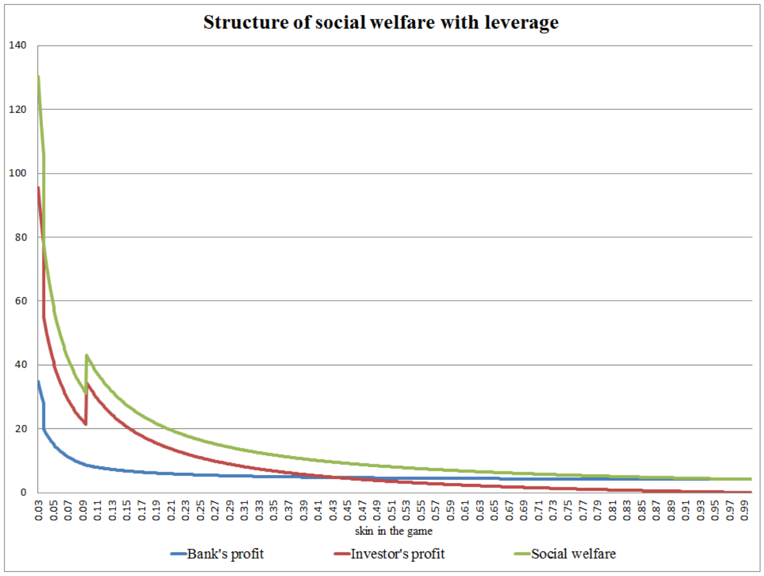

Figure 15. A^L_{1-}-A^L_0-S^L structure

$(X_H = 1.2,\ X_L = 0.8,\ p = 0.55,\ \mathbb{E}(Y) = 1.03,\ q = 0.5,\ \alpha = 0.2,\ \beta_s = 0,\ \beta_a = 0.4,\ h = 0.85)$

Structures with three jumps in social welfare, the A^L_{1+} - S^L - A^L_0 - S^L structure and the A^L_{1-}-S^L-A^L_0-S^L structure, hold if the following conditions are satisfied, respectively:

$$\underline{d} < d^L_{1+} < d^* < d^L_0 \qquad \text{and} \qquad P^* \geq 1, \tag{77}$$

$$\underline{d} < d^L_{1-} < \min\left(d^*, d^{L*}\right) < \max\left(d^*, d^{L*}\right) < d^L_0 \qquad \text{and} \qquad P^* < 1. \tag{78}$$

These conditions, however, do not hold with reasonable parameters.

6.4.2 *Comparative Statics*

Comparative statics and their implications that we investigated in the case without leverage still hold true with leverage as we can see from the following comparative statics:

$$\frac{\partial d^{L*}}{\partial \mathbb{E}(Y)} = \frac{\beta_a P^* \left(1 - \frac{1-h}{h}\frac{1-P^*}{P^*}\right)}{X_H - P^* - (1-\beta_a)[\mathbb{E}(Y)-1]P^*\left(1 - \frac{1-h}{h}\frac{1-P^*}{P^*}\right)} \times \left(1 + \frac{(1-\beta_a)[\mathbb{E}(Y)-1]P^*(1 - \frac{1-h}{h}\frac{1-P^*}{P^*})}{X_H - P^* - (1-\beta_a)[\mathbb{E}(Y)-1](1 - \frac{1-h}{h}\frac{1-P^*}{P^*})P^*}\right) > 0 \tag{79}$$

$$\frac{\partial d^{L*}}{\partial q} = \frac{\beta_a[\mathbb{E}(Y)-1](\overline{P} - X_L)}{X_H - P^* - (1-\beta_a)[\mathbb{E}(Y)-1]P^*(1 - \frac{1-h}{h}\frac{1-P^*}{P^*})} \times \left(\frac{1-h}{h}(1 + \frac{1-P^*}{P^*}) + [1 - \frac{1-h}{h}\frac{1-P^*}{P^*}]\right.$$
$$\left. \times \left\{1 + \frac{[1 + (1-\beta_a)[\mathbb{E}(Y)-1]\frac{1-h}{h}(1 + \frac{1-P^*}{P^*}) + (1-\beta_a)[\mathbb{E}(Y)-1](1 - \frac{1-h}{h}\frac{1-P^*}{P^*})]}{X_H - P^* - (1-\beta_a)[\mathbb{E}(Y)-1]P^*(1 - \frac{1-h}{h}\frac{1-P^*}{P^*})}\right\}\right) > 0. \tag{80}$$

Furthermore, the higher h is, the larger d^{L*} is, as we can verify from the following:

$$\begin{aligned}\frac{\partial d^{L*}}{\partial h} &= \frac{\beta_a[\mathbb{E}(Y)-1]\frac{P^*}{h}(\frac{1-h}{h}\frac{1-P^*}{P^*}+\frac{1-P^*}{P^*})}{X_H-P^*-(1-\beta_a)[\mathbb{E}(Y)-1]P^*(1-\frac{1-h}{h}\frac{1-P^*}{P^*})} \\ &\times(1+\frac{(1-\beta_a)[\mathbb{E}(Y)-1]P^*(1-\frac{1-h}{h}\frac{1-P^*}{P^*})}{X_H-P^*-(1-\beta_a)[\mathbb{E}(Y)-1]P^*(1-\frac{1-h}{h}\frac{1-P^*}{P^*})})>0\end{aligned} \tag{81}$$

This implies that the bank is less likely to utilize the new projects when it has many liabilities because mandatory liquidation curtails the merit of the new investments. We can infer that tight regulation of skin in the game is more likely to result in a loss of social welfare during a recession, especially for highly levered financial intermediaries. Hence, we can address that the countercyclical regulation is more necessary when financial institutions are highly levered.

7. Conclusion

In this paper, we presented a model regarding securitized banking and risk retention requirement. The basic framework is based on Shleifer and Vishny's (2010) work, which focused on the relationship between volatile financial markets and securitization. Yet, we shed light on the possible side effects of the current regulation regarding risk retention. Our model reflects more realities in the sense that it includes risky investment projects and asymmetric information between the bank and investors. One of the most distinctive features of our model is that we introduce another real investment project available stochastically in the interim period, and this additional investment project plays a crucial role in our analysis.

We have asked whether the regulation of skin in the game, stipulated as 5% of credit risks by the Dodd-Frank Act, is efficient enough to revitalize the securitization market and to stabilize the financial market. A bank adopts strategies under asymmetric information to maximize its expected return, and a jump might occur in social welfare as the skin in the game increases. The jump might be either a downward one or an upward one, and both of them could exist at the same time. A downward jump in social welfare implies that the bank does not utilize the profitable investment project fully. Various structures of social welfare are categorized, and the conditions for each type are presented. We have found that the possible problem of the loss in social welfare worsens during a recession. That is, the bank is less likely to utilize new projects fully when expected returns and the availability of them are low. During an economic depression, a threshold of skin in the game regarding a downward jump decreases, and the depth of it increases, while that of the upward jump decreases. These findings shed light on the necessity of regulation that reflects the features of individual intermediary and macroeconomic condition. That is, the current regulation with uniform level of skin in the game is not very efficient for revitalizing the securitization market, and needs to be revised to individual and countercyclical one.

Furthermore, our model can explain what Shleifer and Vishny (2010) asserted (i.e., volatile financial markets) even without assuming investors' sentiment if we consider levered financial intermediaries. If there asymmetric information exists about the return of securitized assets and the availability of new projects, a lemon problem occurs in the market, and the levered bank might undergo mandatory liquidation for a haircut, which makes financial market more volatile.

Acknowledgements

We would like to thank the anonymous reviewer for providing us helpful comments and suggestions. This work has been supported by KAKENHI 22710142, 23310103, 26350424.

References

[1] Adrian, T., & Brunnermeier, M. K. (2011). *CoVaR*. Federal Reserve Bank of New York Staff Reports.

[2] AFME (2011). *Securitisation data report Q4:2011.* Prepared in partnership with SIFMA. Retrieved from http://www.afme.eu/WorkArea/DownloadAsset.aspx?id=5890.

[3] Akerlof, G. A. (1970). The market for lemons: Quality uncertainty and the market mechanism. *The Quarterly Journal of Economics, 84*(3), 488-500.

[4] Batty D. L. (2011). Dodd-frank's requirement of "skin in the game" for asset-backed securities may scalp corporate loan liquidity. *North Carolina Banking Institute Journal, 15*, 13-45.

[5] BIS. (2010, July 16). *Countercyclical capital buffer proposal—Consultative document*. Bank for International Settlements. Retrieved July 16, 2010 from http://www.bis.org/publ/bcbs172.htm

[6] Brunnermeier, M. K., Crocket, A., Goodhart, C., Persaud, A. D., & Shin, H. S. (2009). The fundamental principles of financial regulation. *Geneva Report on the World Economy, 11*. Centre for Economic Policy Research.

[7] DeMarzo, P. M. (2005). The pooling and tranching of securities: A model of informed intermediation. *The Review of Financial Studies, 18*(1), 1-35.

[8] DeMarzo, P. M., & Duffie, D. (1999). A liquidity-based model of security design. *Econometrica, 67*(1), 65-99.

[9] Diamond, D. W., & Rajan, R. G. (2011). Fear of fire sales, illiquidity seeking, and credit freezes. *The Quarterly Journal of Economics, 126*(2), 557-591.

[10] Dugan, J. C. (2010, February 2). *Securitization, 'skin-in-the-game' proposals, and minimum mortgage underwriting standards.* Remarks before the American Securitization Forum. Retrieved February 2, 2010 from http://www.occ.gov/news-issuances/speeches/2010/pub-speech-2010-13.pdf.

[11] Fender, I., & Mitchell, J. (2009). *Incentives and tranche retention in securitisation: A screening model.* Bank for International Settlements (BIS) Working Papers No. 289. Retrieved from http://www.bis.org/publ/work289.htm.

[12] Gennaioli, N., Shleifer, A., & Vishny, R. W. (2010). *Neglected risks, financial innovation, and financial fragility.* NBER Working Paper No. 16068. Retrieved from http://www.nber.org/papers/w16068

[13] Gorton, G., & Souleles, N. S. (2007). Special purpose vehicles and securitization. In M. Carey, & R. M. Stulz (Eds.), *The risks of financial institutions* (pp. 549-602). University of Chicago Press.

[14] Gorton, G. B., & Pennacchi, G. G. (1995). Banks and loan sales: Marketing nonmarketable assets. *Journal of Monetary Economics, 35*(3), 389-411.

[15] Hartman-Glaser, B., Piskorski, T., & Tchistyi, A. (2012). Optimal securitization with moral hazard. *Journal of Financial Economics, 104*(1), 186-202.

[16] IMF. (2009). *Global financial stability report: Navigating the financial challenges ahead.* International Monetary Fund. Retrieved from http://www.imf.org/external/pubs/ft/gfsr/2009/02/.

[17] Kashyap, A. K., Rajan, R. G., & Stein, J. C. (2008). *Rethinking capital regulation.* Working Paper for Federal Reserve Bank of Kansas City Symposium.

[18] Kiff, J., & Kisser, M. (2010). *Asset securitization and optimal retention.* IMF working paper No. 10/74. Retrieved from http://www.imf.org/external/pubs/cat/longres.cfm?sk=23691.0

[19] Kiff, J., & Kisser, M. (2011). *A shot at regulating securitization.* Norwegian School of Economics in its series Discussion Papers No. 2011/7.

[20] Levitin, A. J. (2011). *Skin-in-the-game: Risk retention lessons from credit card securitization.* Georgetown Law and Economics Research Paper No. 11-18.

[21] Malekan, S., & Dionne, G. (2012). *Securitization and optimal retention under moral hazard.* Working paper in HEC Montreal - Department of Finance.

[22] Parlour, C. A., & Plantin, G. (2008). Loan sales and relationship banking. *The Journal of Finance, 63*(3), 1291-1314.

[23] Rajan, R. G. (1992). Insiders and outsiders: The choice between informed and arm's lengh debt. *The Journal of Finance, 47*(4), 1367-1400.

[24] Riddiough, T. J. (1997). Optimal design and governance of asset-backed securities. *Journal of Financial Intermediation, 6*(2), 121-152.

[25] Shleifer, A., & Vishny, R. W. (2010). Unstable banking. *Journal of Financial Economics, 97*(3), 306-318.

[26] Stein, J. C. (2012). *Monetary policy as financial stability regulation. The Quarterly Journal of Economics, 127*(1), 57-95.

[27] US Department of the Treasury. (2009, June 17). *Financial regulatory reform: A new foundation.* Retrieved June 17, 2009 from http://www.treasury.gov/initiatives/Documents/FinalReport_web.pdf.

[28] Wu, H. M., & Guo, G. (2010). *Retention ratio regulation of bank asset securitization.* Working paper in National School of Development, Peking University.

Appendix

A. No Cash Hoarding Condition

A.1 Symmetric Information

A.1.1 *Unlevered Bank*

If information is asymmetric and the bank does not have liabilities, the bank does not hoard any cash if the following holds:

$$\begin{aligned}
&\alpha[\mathbb{E}(X)-1]\frac{E_0-C}{d}+(1-\alpha)[\mathbb{E}(X)-1](E_0-C)\\
&+q\{\beta_s[\mathbb{E}(Y)-1]\frac{\mathbb{E}(X)(E_0-C)+C}{d}+(1-\beta_s)[\mathbb{E}(Y)-1][\mathbb{E}(X)(E_0-C)+C]\}\\
&\leq \alpha[\mathbb{E}(X)-1]\frac{E_0}{d}+(1-\alpha)[\mathbb{E}(X)-1]E_0\\
&+q\{\beta_s[\mathbb{E}(Y)-1]\frac{\mathbb{E}(X)E_0}{d}+(1-\beta_s)[\mathbb{E}(Y)-1]\mathbb{E}(X)E_0\} \qquad (82)
\end{aligned}$$

which is equivalent to the following:

$$\begin{aligned}
&0\leq \alpha[\mathbb{E}(X)-1]\frac{1}{d}+(1-\alpha)[\mathbb{E}(X)-1]\\
&+q\{\beta_s[\mathbb{E}(Y)-1]\frac{\mathbb{E}(X)-1}{d}+(1-\beta_s)[\mathbb{E}(Y)-1](\mathbb{E}(X)-1)\} \qquad (83)
\end{aligned}$$

It is obvious that this condition always holds.

A.1.2 *Levered Bank*

The condition for a levered bank not to hoard any cash at time 0 under symmetric information is as follows:

$$\alpha[\mathbb{E}(X)-1]\frac{\frac{E_0}{h}-C}{d}+(1-\alpha)[\mathbb{E}(X)-1](\frac{E_0}{h}-C)$$

$$+q[\beta_s[\mathbb{E}(Y)-1]\frac{(\frac{E_0}{h}-C)\mathbb{E}(X)+C}{d}+(1-\beta_s)[\mathbb{E}(Y)-1]((\frac{E_0}{h}-C)\mathbb{E}(X)+C)$$

$$-(1-p)\{\beta_s[\mathbb{E}(Y)-1]\frac{X_L\hat{S}_L}{d}+(1-\beta_s)[\mathbb{E}(Y)-1]X_L\hat{S}_L\}]$$

$$\leq \alpha[\mathbb{E}(X)-1]\frac{E_0}{dh}+(1-\alpha)[\mathbb{E}(X)-1]\frac{E_0}{h}$$

$$+q[\beta_s[\mathbb{E}(Y)-1]\frac{\mathbb{E}(X)E_0}{dh}+(1-\beta_s)[\mathbb{E}(Y)-1]\frac{\mathbb{E}(X)E_0}{h}$$

$$-(1-p)\{\beta_s[\mathbb{E}(Y)-1]\frac{X_LS_L}{d}+(1-\beta_s)[\mathbb{E}(Y)-1]X_LS_L\}] \tag{84}$$

where

$$\hat{S}_L=S_L-\frac{1-h}{h}\frac{C}{X_L}$$

$$=\frac{E_0}{h}(\frac{1-h}{h}\frac{1-X_L}{X_L})-\frac{1-h}{h}\frac{C}{X_L}$$

This condition is equivalent to the following:

$$q[\beta_s[\mathbb{E}(Y)-1]\frac{(1-p)\frac{1-h}{h}-[\mathbb{E}(X)-1]}{d}+(1-\beta_s)[\mathbb{E}(Y)-1]\{(1-p)\frac{1-h}{h}-[\mathbb{E}(X)-1]\}]$$

$$\leq \alpha[\mathbb{E}(X)-1]\frac{1}{d}+(1-\alpha)[\mathbb{E}(X)-1] \tag{85}$$

This is always satisfied if the following holds:

$$(1-p)\frac{1-h}{h}\leq \mathbb{E}(X)-1 \tag{86}$$

No cash hoarding does not always hold because the amount of capital available for the new projects decreases due to the mandatory liquidation.

A.2 Asymmetric Information

A.2.1 *Unlevered Bank*

When (8) holds, the condition for an unlevered bank not to hold cash at time 0 under asymmetric information is as follows:

$$\alpha[\mathbb{E}(X)-1]\frac{E_0-C}{d}+(1-\alpha)[\mathbb{E}(X)-1](E_0-C)-T_1(E_0-C)$$
$$+q\{\beta_a[\mathbb{E}(Y)-1]\frac{P^*(E_0-C)+C}{d}+(1-\beta_a)[\mathbb{E}(Y)-1][P^*(E_0-C)+C]\}$$
$$\leq \alpha[\mathbb{E}(X)-1]\frac{E_0}{d}+(1-\alpha)[\mathbb{E}(X)-1]E_0-T_1E_0$$
$$+q\{\beta_a[\mathbb{E}(Y)-1]\frac{P^*E_0}{d}+(1-\beta_a)[\mathbb{E}(Y)-1]P^*E_0\} \tag{87}$$

which is equivalent to the following:

$$T_1\leq \alpha[\mathbb{E}(Y)-1]\frac{1}{d}+(1-\alpha)[\mathbb{E}(Y)-1]+q\{\beta_a[\mathbb{E}(Y)-1]\frac{P^*-1}{d} + (1-\beta_a)[\mathbb{E}(Y)-1](P^*-1)\} \tag{88}$$

This condition is more likely to hold when $P^*\geq 1$.

If (8) does not hold, however, the no cash hoarding condition becomes as follows:

$$\alpha[\mathbb{E}(X)-1]\frac{E_0-C}{d}+(1-\alpha)[\mathbb{E}(X)-1](E_0-C)$$
$$+q(1-p)\{\beta_a[\mathbb{E}(Y)-1]\frac{X_L(E_0-C)+C}{d}+(1-\beta_a)[\mathbb{E}(Y)-1][X_L(E_0-C)+C]\}$$
$$\leq \alpha[\mathbb{E}(X)-1]\frac{E_0}{d}+(1-\alpha)[\mathbb{E}(X)-1]E_0$$
$$+q(1-p)\{\beta_a[\mathbb{E}(Y)-1]\frac{X_LE_0}{d}+(1-\beta_a)[\mathbb{E}(Y)-1]X_LE_0\} \tag{89}$$

which is equivalent to the following:

$$q(1-p)\{\beta_a[\mathbb{E}(Y)-1]\frac{1-X_L}{d}+(1-\beta_a)[\mathbb{E}(Y)-1](1-X_L)\}$$
$$\leq \alpha[\mathbb{E}(X)-1]\frac{1}{d}+(1-\alpha)[\mathbb{E}(X)-1] \tag{90}$$

A.2.2 *Levered Bank*

No cash hoarding condition for levered bank gets more complicated under asymmetric information.

If (8) holds and $P^*\geq 1$ — that is, if the mandatory liquidation never occurs — the condition is as follows:

$$\alpha[\mathbb{E}(X)-1]\frac{\frac{E_0}{h}-C}{d}+(1-\alpha)[\mathbb{E}(X)-1](\frac{E_0}{h}-C)-T_1(\frac{E_0}{h}-C)$$
$$+q\{\beta_a[\mathbb{E}(Y)-1]\frac{P^*(\frac{E_0}{h}-C)+C}{d}+(1-\beta_a)[\mathbb{E}(Y)-1][P^*(\frac{E_0}{h}-C)+C]\}$$
$$\leq \alpha[\mathbb{E}(X)-1]\frac{E_0}{hd}+(1-\alpha)[\mathbb{E}(X)-1]\frac{E_0}{h}-\frac{T_1E_0}{h}$$

$$+q\{\beta_a[\mathbb{E}(Y)-1]\frac{P^*E_0}{hd}+(1-\beta_a)[\mathbb{E}(Y)-1]\frac{P^*E_0}{h}\} \tag{91}$$

which is equivalent to the following:

$$\begin{aligned}T_1 &\leq \alpha[\mathbb{E}(X)-1]\frac{1}{d}+(1-\alpha)[\mathbb{E}(X)-1]\\ &+q\{\beta_a[\mathbb{E}(Y)-1]\frac{P^*-1}{d}+(1-\beta_a)[\mathbb{E}(Y)-1](P^*-1)\}\end{aligned} \tag{92}$$

If the mandatory liquidation occurs for $P^*<1$ and the bank still utilizes the new projects fully since (53) holds, the condition is as follows:

$$\begin{aligned}&\alpha[\mathbb{E}(X)-1]\frac{\frac{E_0}{h}-C}{d}+(1-\alpha)[\mathbb{E}(X)-1](\frac{E_0}{h}-C)-T_1(\frac{E_0}{h}-C)-T_2\hat{S}^*\\ &+q\{\beta_a[\mathbb{E}(Y)-1]\frac{P^*(\frac{E_0}{h}-C-\hat{S}^*+C)}{d}+(1-\beta_a)[\mathbb{E}(Y)-1][P^*(\frac{E_0}{h}-C-\hat{S}^*)+C]\}\\ &\leq \alpha[\mathbb{E}(X)-1]\frac{E_0}{dh}+(1-\alpha)[\mathbb{E}(X)-1]\frac{E_0}{h}-T_1\frac{E_0}{h}-T_2S^*\\ &+q\{\beta_a[\mathbb{E}(Y)-1]\frac{P^*(\frac{E_0}{h}-S^*)}{d}+(1-\beta_a)[\mathbb{E}(Y)-1]P^*(\frac{E_0}{h}-S^*)\}\end{aligned} \tag{93}$$

where

$$\begin{aligned}\hat{S}^* &= S^*-\frac{1-h}{h}\frac{C}{P^*}\\ &=\frac{E_0}{h}(\frac{1-h}{h}\frac{1-P^*}{P^*})-\frac{1-h}{h}\frac{C}{P^*}\end{aligned}$$

which is equivalent to the following:

$$\begin{aligned}&q\{\beta_a[\mathbb{E}(Y)-1]\frac{1-P^*+\frac{1-h}{h}}{d}+(1-\beta_a)[\mathbb{E}(Y)-1](1-P^*+\frac{1-h}{h})\}+T_1+T_2\frac{1-h}{h}\frac{1}{P^*}\\ &\leq \alpha[\mathbb{E}(X)-1]\frac{1}{d}+(1-\alpha)[\mathbb{E}(X)-1]\end{aligned} \tag{94}$$

If (8) does not hold, or (53) does not hold even though (8) holds, the condition is as follows.

$$\begin{aligned}&\alpha[\mathbb{E}(X)-1]\frac{\frac{E_0}{h}-C}{d}+(1-\alpha)[\mathbb{E}(X)-1](\frac{E_0}{h}-C)-T_3\hat{S}_L\\ &+(1-p)q\{\beta_a[\mathbb{E}(Y)-1]\frac{X_L(\frac{E_0}{h}-C-S_L)+C}{d}\\ &+(1-\beta_a)[\mathbb{E}(Y)-1][X_L(\frac{E_0}{h}-C-S_L)+C]\}\\ &\leq \alpha[\mathbb{E}(X)-1]\frac{E_0}{dh}+(1-\alpha)[\mathbb{E}(X)-1]\frac{E_0}{h}-T_3S_L\end{aligned}$$

$$+(1-p)q\{\beta_a[\mathbb{E}(Y)-1]\frac{X_L(\frac{E_0}{h}-S_L)}{d}+(1-\beta_a)[\mathbb{E}(Y)-1]X_L(\frac{E_0}{h}-S_L)\} \tag{95}$$

which is equivalent to the following.

$$T_3\frac{1-h}{h}\frac{1}{X_L}+q(1-p)\{\beta_a[\mathbb{E}(Y)-1]\frac{1-X_L+\frac{1-h}{h}}{d}+(1-\beta_a)[\mathbb{E}(Y)-1](1-X_L+\frac{1-h}{h})\}$$
$$\leq \alpha[\mathbb{E}(X)-1]\frac{1}{d}+(1-\alpha)[\mathbb{E}(X)-1] \tag{96}$$

B. No Default Condition

The bank must have at least L_0 on balance sheet at all times to avoid bankruptcy, even when the low payoff is realized from the projects. For I_X projects, it can be represented as follows:

$$\frac{E_0}{h}X_L \geq \frac{1-h}{h}E_0 \tag{97}$$

which is equivalent to the following:

$$X_L \geq 1-h \tag{98}$$

A similar argument is applied to I_Y projects. The amount of capital invested in the new project depends on the payoff of I_X projects, and it is lower when X_L is realized. Let Y_L denote a lower bound of payoff from I_Y projects. The following is the condition for non-default:

$$\frac{X_L E_0}{h}Y_L \geq \frac{1-h}{h}E_0 \tag{99}$$

which is equivalent to the following:

$$X_L Y_L \geq 1-h \tag{100}$$

In fact, we can presume the situation in which (100) always holds. This Y_L satisfying (100) always exists as we do not impose any restriction on the distribution of Y except that $\mathbb{E}(Y)>1$.

Under asymmetric information, the condition corresponding to (100) is as follows:

$$\frac{P^* E_0}{h}Y_L \geq \frac{1-h}{h}E_0 \tag{101}$$

(101) always holds, much like (100).

Therefore, (98) is a necessary and sufficient condition for the bank not to declare bankruptcy.

C. Straightforward Calculation

C.1 Calculation in (12)

$$q(P^*-P_0)+(1-q)\{p(X_H-P_0)+(1-p)(P^*-P_0)\}$$
$$=P^*-\overline{P}+p(1-q)(X_H-P^*)$$
$$=(1-q)[\mathbb{E}(X)+pX_L-p(1-q)X_L-\overline{P}-pq\overline{P}]$$
$$=(1-q)[\mathbb{E}(X)-\overline{P}-pq(\overline{P}-X_L)]$$

$$= (1-\alpha)[\mathbb{E}(X)-1] - q[\{1-p(1-q)\}(1-\alpha)[\mathbb{E}(X)-1] + p(1-q)[\mathbb{E}(X)-X_L]]$$
$$= (1-\alpha)[\mathbb{E}(X)-1] - T_1 \tag{102}$$

where

$$T_1 := q[\{1-p(1-q)\}(1-\alpha)[\mathbb{E}(X)-1] + p(1-q)[\mathbb{E}(X)-X_L]]$$

C.2 Calculation in (54)

$$q[(P^*-P_0)\frac{E_0}{h}] + (1-q)[p\{(X_H-P_0)(\frac{E_0}{h}-S^*) + (P^*-P_0)S^*\} + (1-p)\{(P^*-P_0)\frac{E_0}{h}\}]$$
$$= (P^*-\overline{P})\frac{E_0}{h} + p(1-q)(X_H-P^*)(\frac{E_0}{h}-S^*)$$
$$= [P^*-P_0+p(1-q)(X_H-P^*)]\frac{E_0}{h} - p(1-q)(X_H-P^*)S^* \tag{103}$$

C.2 Calculation in (56)

$$X_H - P^* = X_H - [q\overline{P} + (1-q)X_L]$$
$$= q(X_H+X_L) + (1-q)X_H - q\{\mathbb{E}(X) - (1-\alpha)[\mathbb{E}(X)-1]\} - X_L$$
$$= q\{(1-p)X_H + pX_L\} + (1-q)X_H + q(1-\alpha)[\mathbb{E}(X)-1] - X_L$$
$$= -pqX_H + pqX_L + X_H - X_L + q(1-\alpha)[\mathbb{E}(X)-1]$$
$$= q(1-\alpha)[\mathbb{E}(X)-1] + (1-pq)(X_H-X_L) \tag{104}$$